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Abstract

Innovations are widely accepted as fundamental drivers of economic growth by increasing productivity and creating new markets. However, empirical evidence on the long-term relationship between technological progress and economic growth remains scarce, with few studies considering shifts in technologies' fundamental properties, such as their degree of complexity. Yet, higher levels of complexity are argued to increase technologies' economic potential, and consequently, ignoring this dimension of technologies provides an incomplete picture of innovations' growth effects.

We address this research gap by exploring the relationship between economic growth and technological complexity over more than 170 years in the United States (US). Utilizing patent data, the concept of the complexity frontier, and partial wavelet analysis, we find that economic growth has not been driven by patented innovation and technological complexity for most of this period. However, since the beginning of the ICT revolution in the 1990s, it has significantly contributed to GDP growth.

One Sentence Summary: Technological complexity drives economic growth

Keywords: Innovation, Economic Growth, Technological Complexity, USA, Complexity Frontier, Wavelt Analysis

JEL: O30, O47, N10

1. Introduction

Technological advancement and innovation are widely accepted as fundamental sources of economic growth (Lucas, 1988; Nelson and Winter, 1982; Romer, 1990). New technologies boost the efficiency of established production processes and form the basis for many new products with comparatively high levels of value-added (Gordon, 2016; Romer, 1990; Solow, 1956). Innovation processes are cumulative and combinatorial, implying that new ideas build on existing knowledge and novelty is created through the recombination of existing technological capabilities (Aunger, 2010; Howitt, 1999; Nelson and Winter, 1982).[1]

Many classifications have been proposed, suggesting that specific capabilities are more economically valuable than others, capturing the heterogeneity of technological capabilities. For instance, scholars have argued that capabilities, including large elements of tacit knowledge, are vital for economic success (Cowan et al., 2000; Grimaldi and Torrisi, 2001; Polanyi, 1966). In more recent years, attention has shifted to complexity, where it is argued that the capabilities to develop and use complex activities are at the core of (economic) competitive advantages (Fleming and Sorenson, 2001; Hidalgo and Hausmann, 2009; Kogut and Zander, 1992). This is because, on one hand, it is difficult to learn and imitate these capabilities. On the other hand, they are essential for producing highly valuable goods and services and provide the basis to acquire capabilities in even more complex activities.

However, the link between complexity and economic growth is not straightforward. Developing capabilities in complex domains requires higher R&D efforts compared to simpler ones. This may reduce these competencies' total economic benefits (Broekel, 2019; Mewes and Broekel, 2022). They are also less likely to diffuse, implying that few actors and locations reap their benefits (Balland and Rigby, 2017; Balland et al., 2020), which may lower their overall contribution to growth. Nevertheless, the growth-enhancing effects of capabilities in complex domains find increasing empirical support, with several distinct empirical foci. Some studies concentrate on quantifying economies' latent holistic capabilities to produce complex goods and services, known as economic complexity, and confirm its role in stimulating economic growth in countries (Hausmann et al., 2013; Hidalgo et al., 2009) and regions (Chávez et al., 2017; Pérez-Balsalobre et al., 2019). Another set of studies focuses on technological complexity, capturing the capabilities to invent and use higher levels of sophisticated technological knowledge. Similar to economic complexity, several studies show its positive effect on economic growth (Mewes and Broekel, 2022; Rigby et al., 2022). However, some studies also report a negative relationship between these types of capabilities and growth (Antonelli et al., 2020).

While most studies acknowledge that complexity primarily determines long-term economic growth, the analyzed periods rarely span more than two decades. The study by Sweet and Eterovic (2019) is a notable exception as it considers four decades. Moreover, few studies discuss or evaluate the interwoven and potentially bidirectional relationship between complexity and economic growth.

The present study addresses these gaps. Utilizing patent data and the concept of the so-called complexity frontier proposed by Mewes and Broekel (2022), we empirically capture the capabilities of the US economy to invent and apply the most complex technologies. Secondly, we relate shifts of the frontier over more than 170 years to national economic growth using wavelet

[1] Knowledge is to be understood very broadly including ideas, understandings, codified information, and artifacts.

gain analysis, which allows a Granger-causal identification of the link between complexity and economic growth, as well as vice versa (Aguiar-Conraria et al., 2013, 2012; Klarl, 2016; Verona, 2020).

The US complexity frontier has moved upwards over time, indicating that the capabilities to invent and employ increasingly sophisticated technologies have been growing continuously. Additionally, the composition of the frontier illustrates how major technology groups rise and fall in their importance in shaping the complexity landscape over time.

Crucially, our results suggest that for the longest time, other factors than the capabilities in the most complex technologies were more important for the growth of the US economy. Between 1890-1907, economic growth drove the development of the complexity frontier instead of vice versa. However, from the 1970s onward, the picture has changed, and we find strong evidence that capabilities in the most complex domains contributed to economic growth. Accordingly, economic development seems to have entered the age of complexity, which has significant implications for innovation and R&D policy.

The paper is structured as follows. First, the presentation of the theoretical basis highlights how capabilities in technologically complex domains can enhance or reduce economic growth. Section 3 introduces the empirical approach to analyzing the relationship between technological complexity and economic growth. Subsequently, our results are presented and discussed in Section 4. Section 5 concludes the paper with some final reflections and an evaluation of the study’s limitations.

2. Theory

2.1. A new perspective on complexity

There is little doubt about technological progress being the major driver of long-term economic growth (Aghion and Howitt, 1992; Nelson and Winter, 1982; Romer, 1990). Technological development has greatly enhanced people's economic prosperity by expanding access to resources and energy, improving the efficiency of production and services, and enlarging the portfolio, functionality, and usefulness of products and services. Yet, technological progress is not homogeneous, nor does it happen in a monotonic fashion. Its intensity and impact vary over time, location, and technology (Aghion et al., 2009; Carlino et al., 2007; Kerr, 2009; Kondratiev, 1926; Šmihula and Von, 2010). Some of these variations have been attributed to breakthrough (Phene et al., 2006), radical (Kline and Rosenberg, 1986), and atypical innovation (Uzzi et al., 2013), as well as to innovation in key-enabling (Evangelista et al., 2018) and general-purpose technologies (Bresnahan and Trajtenberg, 1995). Despite fundamental differences in these conceptualizations, they all emphasize that it is not just the speed and magnitude of technological progress in general that shapes economic development but that it matters in which economic and technological domains innovations occur.

In this context, the differentiation between simple and complex domains has recently (re)gained the attention of scholars. The idea that economic development is related to the sophistication of products and technologies employed in an economy has been around for some time (see, e.g., Antonelli, 1995; Fleming and Sorenson, 2001; and the review by Arthur, 2021). However, it was

Hidalgo et al. (2009)'s introduction of "economic complexity" that promoted the view of a stronger causal link from (economic) complexity to economic growth:

> "*A possible explanation for the connection between economic complexity and growth is that countries that are below the income expected from their capability endowment have yet to develop all of the products that are feasible with their existing capabilities. We can expect such countries to be able to grow more quickly, relative to those countries that can only grow by accumulating new capabilities.*" (Hidalgo et al., 2009, p. 10575)

In other words, countries with capabilities in economic activities that are relatively more complex than those in which other countries at similar levels of economic development have specialized can be expected to outgrow these in the future. The authors also introduce a method to empirically capture the complexity of such capabilities (the so-called method of reflection) and provide empirical evidence that these:

> "*(ii) are strongly correlated with income per capita; (iii) are predictive of future growth; and (iv) are predictive of the complexity of a country's future exports, making a strong empirical case that the level of development is indeed associated to the complexity of a country's economy."* (Hidalgo et al., 2009, p. 10575)

Their work identifies and quantifies economies' (latent) capability to competitively produce complex products and services. It also establishes a direct link between complexity and economic growth by associating improvements in economic complexity with structural transformations from capabilities in relatively simple, low-tech domains to more advanced ones. Capabilities in more advanced domains are equated with higher productivity, implying that economic growth stems from economies moving from low- to high-productivity activities (Balland et al., 2022). Like traditional economic growth theories, economic complexity sees knowledge and the capabilities to do certain things as core ingredients for economic growth. However, while endogenous growth theory stresses the shareable aspect of knowledge (non-rival nature), economic complexity focuses on the unique, specialized nature of knowledge and capabilities, or, as Hidalgo (2023) puts it, their "*non-fungibility."* To describe economies' capabilities to produce complex products and services, economic complexity aggregates knowledge and capabilities across various dimensions, including technology, infrastructure, and institutions (Hidalgo et al., 2009). This is one of its (empirical) strengths, as it can capture economies' "*latent*" capabilities by providing a kind of "*propensity scores for systems* [to experience economic growth] *where the exact factors and their combinations are unknown*" (Hidalgo, 2023, p. 4).

However, this strength comes with a "*catch-all*" perspective that keeps specific capabilities and their relative importance latently hidden in theory and empirics. This can be desirable in certain contexts, e.g., policy advice. In other contexts, it is not helpful, e.g., for understanding the causal forces underlying economic growth. The present paper is interested in the latter. It seeks to dissect the growth contributions of specific capabilities, particularly the ability to invent and use complex technologies - a factor argued to drive economic growth. Complex technologies are defined here as those in domains with intricate systems and advanced knowledge, requiring extensive expertise, interdisciplinary cooperation, and significant resources. Our work aligns with Mewes and Broekel (2022) in examining how technological complexity, a well-established topic in economics (see, e.g., Fleming and Sorenson, 2001; Saviotti, 2011), connects to economic expansion. Authors like Balland and Rigby (2017), Crespo et al. (2017), and Mewes and Broekel (2022) have highlighted this connection, suggesting that economies adept at harnessing complex technologies gain competitive edges and access to higher growth trajectories. We explore this assertion in greater depth in the following discussion.

2.2. Technological complexity stimulates economic growth

The link between capabilities in complex domains and economic growth is grounded in Hidalgo et al.'s (2009) theory, which states that countries with income below their capability potential have yet to utilize their existing capabilities fully. Such countries are poised for rapid growth by developing new products that their existing capabilities make possible. We extrapolate this argument to the realm of complex technological capabilities, suggesting that harnessing these underutilized capacities can similarly spur economic growth. Economies may show a mismatch between their actual economic performance (e.g., GDP) and their inherent capabilities. In other words, there are situations when an economy possesses the capabilities to produce more or to create more valuable outputs that are not yet actualized. This may occur because products and services arising from complex technological capabilities are underdeveloped or diffused slowly. In such instances, we anticipate that the economy will grow as it begins to capitalize on these untapped potentials.

We highlight three main "growth-enhancing" channels through which advanced technological capabilities may drive economic expansion.

The first is the differences in economic competition between products and services based on complex and simple technologies. Goods and services whose production requires capabilities in complex technologies tend to be characterized by lower competitive pressure, allowing their producers to extract higher economic rents, stimulating further investments, and ultimately leading to higher growth (Mewes and Broekel, 2022). Developing and applying complex technologies are resource-intensive and costlier than simpler technologies (Galbraith, 1990; Pintea and Thompson, 2007; Zander and Kogut, 1995). Typically, advancing complex technologies takes more time, implies higher rates of failure and the utilization of more costly infrastructure, as well as the integration of a greater variety of specialized expertise and reliance on collaboration (Cohen, 2010; Galende, 2006; Mastrogiorgio and Gilsing, 2016; Powell and Giannella, 2010; Rivkin, 2000; Stephan, 2010). Fewer actors are equipped and inclined to undertake these riskier investments, resulting in less competition than in simpler technological areas (Teece, 1986). This is further strengthened by competitive advantages in complex technologies being less likely threatened by imitation (Cohen, 2010). In sum, capabilities in more complex technologies are associated with lower competition. This may stimulate economic growth through producers accumulating larger economic rents that are partly (re-) invested in further expansion.

The second growth-enhancing effect of complex technologies stems from their ability to solve more sophisticated problems and fulfill advanced needs, providing larger functionalities and more flexibility than simpler technologies. Products and services developed using such technologies are often more capable and versatile, as seen in the literature (Griffin, 1997; Valverde and Sole, 2015). Simply put, the more complex the technology, the more advanced the product, and typically, the greater the utility. Consider the smartphone as an example; it combines communication, entertainment, and information into one device, offering more use than devices dedicated to single functions. Its multifunctionality and advanced features translate to higher consumer value, prompting a willingness to pay more. Artificial Intelligence (AI) is another case in point. The complex technologies it relies on, like machine learning and neural networks, lead to a product that's incredibly adaptable across different problems. Moreover, such products usually combine multiple complex technologies, which means the complexity of one technology is often linked to and increased by its integration with other (complex) technologies (Gambardella et al., 2021). Complex technology capabilities' versatile and transferable nature spurs innovation and growth across different sectors. For example, electronics miniaturization, perfected for smartphones, is now used in medical devices, space exploration, and IoT devices. Machine learning algorithms have gone from powering search engines to aiding in medical diagnoses and driving autonomous vehicles. In summary, having the know-how in complex technologies leads to creating more useful and, thereby, more valuable products. At the same time, this know-how is applicable to a broader set of problems, generating economies of scale and scope. These wider applications and re-uses of technology and the higher value of the products contribute to overall economic growth (Hidalgo et al. 2009). Additionally, some complex technologies allow for more resource-efficient production methods, adding another layer of growth impact (Zou et al., 2022).

The third source from which capabilities in complex technologies contribute to economic growth draws from Hidalgo et al.'s (2009) perspective, which posits that successfully mastering complex technologies in the past positions actors better to make further advances. This premise is underpinned by the notion that knowledge acquisition and learning are inherently cumulative. Skills and insights gained from developing one complex technology can be applied and extended to learn and create even more sophisticated technologies in the future (Balland and Rigby, 2017; Balland et al., 2020; Mewes and Broekel, 2022).

Beyond the straightforward concept of cumulative learning, another way in which acquiring capabilities in complex technologies contributes to growth mirrors the principles of the dynamic capabilities framework (Katkalo et al., 2010; Teece et al., 1997). According to this framework, mastering complex technologies equips actors with specific skills, knowledge, institutions, and infrastructure that jointly form an adaptive and responsive capability, enabling them to identify and acquire new and adjust old skills quicker and more efficiently. This ability to continuously adapt and reconfigure resources and capabilities in response to changing environments is crucial for sustained competitive advantage and economic growth. Expanding on this argument, mastering one complex technology creates a foundation for innovation, as it often requires the development of new processes, tools, and collaborative networks. This foundation provides a fertile ground for further technological advancements. As actors innovate and solve problems within one domain, they generate a ripple effect of knowledge and skill enhancement propagating through industries and sectors. For instance, early developers of computer chips now lead in advanced semiconductor technologies, showing how initial capabilities can advance. This evolution reflects a virtuous cycle where each technological achievement paves the way for the next, propelling economic growth through a sustained innovation and skill development trajectory (Dosi, 1988; Hidalgo, 2015; Teece, 1988; Teece et al., 1997).

Some arguments contrast with these three sources for capabilities in complex technologies fostering economic growth. Most importantly, there are ever-increasing research efforts. Between 1953 and 2017, the total R&D spending in the US grew from about 5 billion US dollars to more than 656 billion US dollars, i.e., more than tenfold (AAAS, 2022). While the output (patents and publications) has substantially risen as well, Schmookler (1966) and Strumsky et al. (2010) document a substantial decrease in R&D productivity, something Bloom et al. (2017) later confirmed. Strumsky et al. (2010) argue that this decreasing productivity is related to technological complexity:

> "*...research problems over time grow increasingly esoteric and intractable. Innovation, therefore, grows increasingly complex, and correspondingly more costly*." (Strumsky et al., 2010, p. 497)

The underlying mechanism is not only that research plucks the "lowest fruit" first (Strumsky et al., 2010), but also that complex technologies are characterized by more extensive and more diverse sets of distinct (knowledge) components that are extensively interrelated in heterogeneous ways. Innovation is, therefore, more difficult in more complex technologies because their greater numbers of different components and interrelations allow for an exponentially larger number of combinations that do not work, which must be eliminated through research (Fleming and Sorenson, 2001). Hence, the costs of the discovery process increase disproportionately with complexity. This implies that when all technologies become more complex on average, the same rate of inventions can only be realized with growing R&D investments (Rescher and Michalos, 1979; Strumsky et al., 2010). In addition to the greater magnitude of resources required for their discovery, more complex technologies also demand greater management capabilities for their application, which must be acquired through costly path-dependent learning and experiences over long periods (Dosi, 1988; Nelson and Winter, 1982).

Another reason for the potential growth-hampering effects of (rising) technological complexity is their potentially lower diffusion speed. As mentioned above, acquiring the skills to learn, master, or even replicate complex technologies is a formidable task, often achievable by only a select few. This is because it necessitates specialized and related resources and capabilities that are not widely available and are expensive to develop. Consequently, this limitation hinders the spread of complex technologies more so than simpler ones and intensifies the obstacle of geographic distance (Balland and Rigby, 2017). The broader economic impact of technologies is more pronounced when they are adopted by more actors who, in doing so, enhance their capabilities. Therefore, complex technologies' slower and spatially constrained dissemination could significantly dampen their potential to stimulate economic growth. Furthermore, the constrained diffusion of these technologies and their potent local effects on economic development will likely exacerbate economic disparities. Regions that successfully harness complex technologies can surge ahead, reaping the benefits of innovation and productivity gains. In contrast, regions that lag in adopting these technologies fall further behind, widening the gap. This divergence can fuel a cycle of increasing economic inequality as the benefits of technological advancements become concentrated in areas already equipped with the necessary infrastructure and skilled workforce (Pinheiro et al., 2022), many of which tend to be urban regions (Broekel et al., 2023).

In sum, significant arguments challenge the potentially growth-enhancing effects of capabilities in complex technologies. While the positive relationship between capabilities in complex economic domains and economic growth is supported by more and more empirical evidence (Albeaik et al., 2017; Gala et al., 2018; Hidalgo et al., 2009), a picture is only slowly emerging concerning capabilities in complex technologies. Mewes and Broekel (2020) recently analyzed European regions and identified a positive contribution of these capabilities to economic growth. Li and Rigby (2022) add further support, showing that Chinese regions diversifying their capabilities into more complex technologies experience higher growth. Both studies focus on the most recent times and cover periods of 15 and 25 years, respectively. Whether the identified positive relationship has been a shaping force in long-term economic development remains to be determined. This motivates the present paper and its central hypothesis:

> **H1:** The capabilities to advance and use technological complexity stimulate economic growth.

To test this hypothesis, we assess the relationship over the last 170 years for the USA. During this time, the US economy has undergone substantial structural shifts, moving from a primarily agrarian to a manufacturing-based and eventually to a service-based economy (Iscan, 2010). Moreover, until the end of the 19th century and the beginning of the 20th, much economic growth was clearly based on increasing capital, natural resources, and labor (Abramovitz and David, 1999; Mowery, 2010). Consequently, capabilities in complex technologies likely played a less critical role. In addition, the knowledge and innovation-generation process evolved massively during this long time span. For instance, only some inventors had formal technical qualifications in the middle of the 19th century (Khan, 2006). Most primarily worked by themselves. Professional research labs and collaboration as central elements of modern R&D only became relevant in the middle of the 20th century in response to the growing complexity (van der Wouden, 2020). We, therefore, expect that the relationship between the capabilities in complex technologies and economic growth is not constant over time but rather underwent substantial reconfigurations.

> **H2a:** The relationship between capabilities to advance and use complex technologies and economic growth is time-variant.
>
> **H2b:** The positive link between the capabilities to advance and use complex technologies and economic growth (H1) is strong in more recent periods.

We will test the hypotheses using a novel dataset outlined in the following section, covering technology complexity and key aspects of the US economy over 170 years.

3. Empirical approach

3.1. Patent data and the complexity frontier

To start our analysis, we assess the complexity of technologies by using patent data from the United States Patent and Trademark Office (USPTO). The pros and cons of using patent data to approximate technological competencies, progress, and innovation are extensively debated (Griliches, 1990). The most important deficits are that it does not give any insights into non-patented innovation and that it does not capture the actual application of technologies. Nevertheless, patents are the only large-scale data source providing detailed information about technological knowledge over a long time span. In this study, we use the information on all utility patents granted between 1836 and 2016 with at least one inventor from the US. Each patent is assigned to one or multiple Cooperative Patent Classification (CPC) technology classes. The hierarchical CPC scheme is our basis for distinguishing technologies at the four-digit level, a common approach in the literature (Broekel, 2019; Buarque et al., 2020). We exclude all patents associated with the "Y" CPC classes, as they comprise patents not clearly classified according to technological properties, which reduces the sample to 5,646,235 patents and 654 technology classes.

There is no standard way of assessing the complexity of technologies or patents. Multiple approaches are available. Most frequently, scholars rely on an adaptation of the economic complexity approach by (Hidalgo et al., 2009) to patent data to calculate a complexity index at the level of patent technology classes (Balland and Rigby, 2017). Another prominent approach is Fleming and Sorenson's (2001) N/K complexity measure at the level of individual patents (Balland et al., 2020; Sorenson et al., 2006; van der Wouden, 2020). However, recent assessments of these measures by Broekel (2019) and Pintar and Essletzbichler (2022) revealed some problematic properties of the resulting indices, including indices based on the economic complexity framework being unstable over time and older technologies being assessed as more complex in the case of the N/K measure. Furthermore, studies at the regional level failed to establish a positive association of both with economic growth for both approaches (Antonelli et al., 2020; Mewes and Broekel, 2022). Given these issues, we use the structural diversity measure proposed by Broekel (2019). Applied to patent data, unlike the other two measures above, it has been shown to resemble basic characteristics usually associated with technological complexity (Broekel, 2019), is relatively stable over time (Pintar and Essletzbichler, 2022), and has been used by Mewes and Broekel (2022) to confirm a positive relationship between territories' capabilities in complex technologies and their economic growth.

In essence, Broekel (2019) contends that complexity emerges from the heterogeneity and integration of knowledge domains. Consider the smartphone as an example: ostensibly a communication device, it represents a nexus of knowledge from computer science, electrical engineering, material science, and optics for features like the camera. It also encompasses insights from psychology and ergonomics to enhance user interface design. While the principles within each domain may be straightforward in isolation, their convergence within smartphone development introduces significant complexity. This is attributable not solely to the variety of domains involved but to the intricacies of fusing them into a seamless, functional entity.

Therefore, a patent focusing on a single element, such as touch screen technology, might exhibit lower complexity than one encapsulating the collective integration necessary for a fully operational smartphone. Lobo et al. (2010) suggest using the number of patent classes on patents as a rudimentary measure of technological complexity. However, this approach falls short of capturing the comprehensive expertise and the synthesis challenge of these diverse fields within an entire technology. A more nuanced conceptualization of complexity is captured in the measure developed by Broekel (2019) that considers the heterogeneity of the knowledge required in all applications of a specific technology and the variations in how this heterogeneity must be integrated.

Building on this, we employ the two-step approach by Mewes and Broekel (2022) to approximate an economy's capabilities in inventing, learning, and mastering complex technologies. The first step involves quantifying the complexity of each technology class (technology in the following) in a given year. The second step synthesizes this data into a singular index representing the economy's capacity to invent and utilize complex technologies effectively, which is labeled the "*complexity frontier*".[2]

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[2] More formally, the complexity frontier represents the capabilities in *the most* complex technologies. Nevertheless, given that this frontier transcends the confines of individual technologies and may encompass only segments of the patents related to a particular technology, we avoid using the term 'the most' to describe these capabilities.

Calculation of complexity: For each of the 654 technologies (i) and each year, the index value of structural complexity is calculated based on its so-called combinatorial network (Broekel, 2019). This network resembles the idea of technologies being systems of knowledge elements (the nodes) that are interrelated (the edges) to fulfill the technology's purpose. In general, technologies that feature many distinct elements (a large number of nodes) that are intensively interconnected (edges) tend to be more complex (Simon, 1962). However, Broekel (2019) points out that these networks are additionally characterized by multiple distinct network (sub-)topologies (e.g., stars, lines, circles, lattices, etc.). Networks with more distinct topologies contain more information, making them harder to invent, learn, and utilize, all of which represent characteristics of more complex technologies. A quantification of this diversity of topologies in technologies' combinatorial networks, which is positively correlated to the number of nodes and edges, can thus be used as a complexity index. Broekel (2019) employs the network diversity score (Emmert-Streib and Dehmer, 2012) to obtain such an index and shows that the resulting values of technological complexity mirror many stylized facts associated with technological complexity.

In our methodology, we develop a combinatorial network for each technology (classified by 4-digit CPC codes), where the connections between patent classes (10-digit CPC codes) that co-occur on patents are indicative of links (combinations) among patent classes, serving as proxies for the technology's (knowledge) elements. Importantly, these elements extend beyond the 10-digit CPC codes within the primary 4-digit CPC category to encompass all that co-occur on patents associated with the focal technology. This approach implicitly accounts for the complexity of a technology, which may stem from the need to integrate it with other technological domains.

To account for the fluctuating nature of technology-specific patent numbers, we apply a three-year moving window approach whereby all patents assigned to the focal patent class between *t-2* and *t* are considered in the construction of the combinatorial network at time *t*. The resulting undirected network is binarized, and a random sample of sub-networks G_M is drawn from it using a walk-trap approach. For each sampled network, an individual diversity score ($iNDS_{i,c}$) is calculated as $\frac{\alpha_{module} * \gamma_{graphlet}}{\theta_{module} * \theta_{\lambda}}$, with α_{module} being the share of modules (the number of modules M divided by the number of nodes n), $\gamma_{graphlet}$ being the ratio between the numbers of graphlets of size three and four, θ_{module} being the variance of module sizes m and θ_{λ} is the Laplacian (L) matrix's variability. The final SD_i is the average of the $iNDS_{i,c}$ across the sample of networks G_M calculated as $\frac{1}{S}\sum_{G_c \in G_M}^{S} iNDS_{C,i}$. Crucially, the central elements of the network diversity score by Emmert-Streib and Dehmer (2012) resemble properties of complex networks. However, its precise definition is the outcome of numerical optimization to maximize differentiation between simple, ordered, and complex networks.

The value SD_i is multiplied by minus one and taken in logs to obtain an index with easily readable values and larger values indicating higher degrees of complexity (Broekel, 2019; Emmert-Streib and Dehmer, 2012). At the end of the first step, 654 annual values of technological complexity are obtained.

In the second step, we calculate the so-called complexity frontier (CF), which was implicitly introduced by Mewes and Broekel (2022). To describe an economy's capabilities in complex technologies at a certain moment, these authors argue that relying on the (weighted) average across all technologies it possesses capabilities in will give a misleading picture. Such reliance would diminish an economy's competencies in complex technologies when it is simultaneously specialized in simple ones. Therefore, the authors suggest assessing the highest level of technological complexity that an economy can advance and manage, which we term the "complexity frontier." This frontier represents the maximum complexity that can potentially be incorporated into new products and services. Consequently, advancing this frontier creates opportunities for more sophisticated products and services expected to drive economic growth.

In practice, we construct the complexity frontier of the US economy by ordering all occurrences of four-digit CPC classes (i) on the patent applications of a specific year (t) according to their value of structural diversity ($SD_{i,t}$). When patent classes appear on multiple patents, they are listed multiple times. The resulting vector M_t can be thought of as all utilization instances of all individual technologies (four-digit CPC classes) in invention processes in year t sorted by technologies' levels. Multiple instances of the same technology on the same patent are disregarded, implying that all unique appearances of technologies on patents are equally weighted. Subsequently, the vector is sorted in descending order, giving the ranked complexity distribution of technologies' usages. On this basis, the annual values of the complexity frontier value (CF_t) are calculated as the median complexity value of the x-percent most complex occurrences ($M_{t,x}$). Figure 1 visualizes the approach. In the following, we focus on the 5% percentile ($CF5_t$) whereby alternative specifications are used as robustness checks. The values of CF5 represent our main explanatory variable.

$$CF_{t,x} = Median(0.9 + 0.9 + 0.8) = 0.9$$

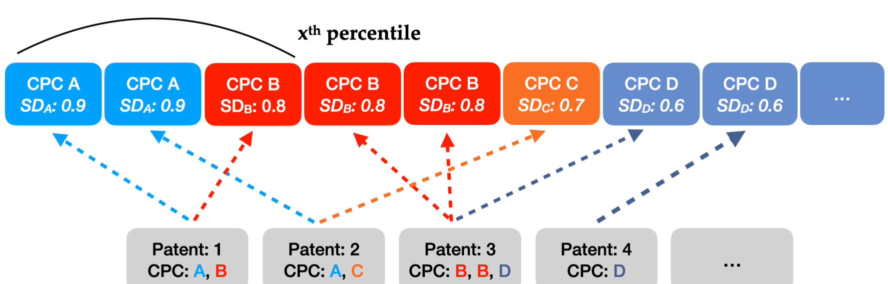

Figure 1: Construction of the complexity frontier

3.2. Additional data

To identify the relationship between the US economy's competencies in complex technologies (complexity frontier) and its economic growth, we need to approximate the latter over 170 years and consider alternative confounders.

Our dependent variable is annual GDP growth estimated with information on the annual US Gross Domestic Product per capita (real GDP per capita (*cgdppc* series), GDP/c in the following). We obtained the data from the Maddison Historical Statistics Project Database 2018, which is identical to the Penn World tables (Bolt et al., 2018). This data is adjusted for price effects and provides comparable values from 1836 to 2016.

The most crucial confounder variable is the intensity of inventive activities, which we approximate with the total number of patents awarded annually. In addition to allowing for differentiation between the quantity and the quality (complexity) dimension of inventions, it can also be seen as a (rough) proxy for total R&D efforts and potential size effects in the construction of the complexity variable (Broekel, 2019). Nevertheless, it shares all the problems of using patent data discussed above.

To further isolate the effects of complexity on economic growth, we also consider some of the most important trends that have shaped aggregate economic growth during this time. These include the growth of the total population, shifts in the share of service employment, changes in the average years of schooling, and variations in the share of the urban population.

The total population per year is obtained from the Penn World tables (Bolt et al., 2018). The increasing tertiarization of the economy, which shapes its growth path (Teixeira and Queirós, 2016), is approximated with the employment share of service employment. This data is available from 1869 to 2005 from the Herrendorf et al. (2014) dataset (Herrendorf et al., 2014). However, until 1920, values are provided only for the end of the respective decade. To obtain the necessary annual values, we rely on the Compound Annual Growth Rate (CAGR) approach and calculate the missing annual values between 1869 and 1920 based on the information for 1881, 1891, 1901, 1911, and 1920. For each decade, the annual growth rate is calculated by $g_d = \frac{Y_{end}}{Y_{beginn}}^{\frac{1}{10}} - 1$. The values for individual years are defined by $Y_{t,t=]begin,end[} = Y_{beginn} * (1 + g_d^{t-beginn})$. For the time after 2005, the share of service employment is taken from the Human Development Reports data.[3] The period 2005-2010 lacks annual information. Hence, the missing values are calculated with the CAGR approach using the information for 2005 and 2010. From 2010 to 2016, this source provides annual data.

Another critical factor is the population's general level of education (Lee and Lee, 2016; Romer, 1990), which we capture by the average years of schooling for the population 15-64 years. The data comes from the Barro-Lee dataset (Lee and Lee, 2016).[4] It contains values for five-year periods from 1870 to 1990. Again, we calculate corresponding annual values using the Compound Annual Growth Rate (CAGR) approach for the missing individual years. Annual values for years after 1990 are extracted from the Human Development Reports.[5]

Lastly, we consider the degree of urbanization, which is widely seen as a crucial driver of creative activities and economic growth (Balland et al., 2020; Glaeser, 2011; Youn et al., 2016). We use the share of the US population living in urbanized regions from Our World in Data.[6] This source provides values for the full decades from 1790 to 1950. Annual values from 1869 to 1950 are estimated as outlined above. From 1950 to 2016, annual values are available. Figure 2 gives insights into the correlation structure, and Table 1 presents some basic descriptives.

Clearly, other factors have played decisive roles in the development of the US economy, including the growing capital intensities, actual R&D investments, economic policies, westward expansion, and natural resources. However, no consistent time series data is available for the studied period. While many of these factors likely correlate to the considered control variables, identifying these confounders remains limited. Therefore, we employ specific wavelet tools to address this issue.

[3] http://hdr.undp.org/en/indicators/150706 (10.08.2022).
[4] https://barrolee.github.io/BarroLeeDataSet/LeeLee/LeeLee_attain_MF1564.xls (10.08.2022).
[5] http://hdr.undp.org/en/indicators/103006 (10.08.2022).
[6] https://ourworldindata.org/grapher/urbanization-last-500-years (10.08.2022).

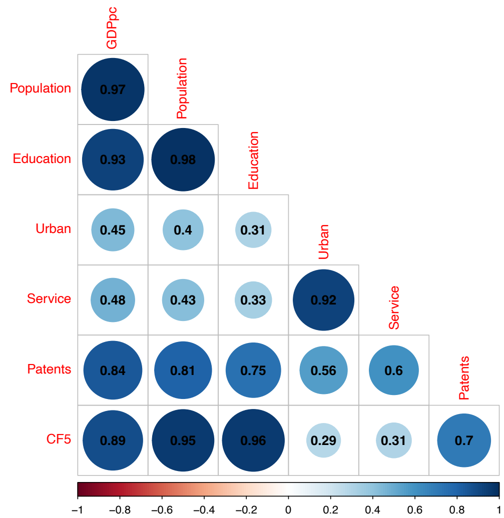

Figure 2: Correlation of core variables

	n	*min*	*max*	*range*	*median*	*mean*	*var*	*std.dev*
year	184	1836	2019	183	1,927.5	1,927.5	2,836.7	53.3
GDPpc	181	2,507	53,015	50,508	8,850	16,226.2	227,148,698	15,071.5
Population	181	157,53000	324,656,000	308,903,000	117,857,000	133,568,193	8.3705E+15	9,149,0346.4
Education	153	1.1	13.4	12.3	8.7	8.9	9.4	3.1
Urban	153	0.1	82.3	82.2	0.6	4.8	327.6	18.1
Service	154	0.2	79.4	79.2	0.5	5.6	375.4	19.4
Patents	182	2	151,108	151,106	23,757	30,464.6	82,102,3583	28,653.5
CF5	180	2.5	13	10.5	8.2	8.7	8.2	2.9

Table 1: Descriptives

3.3. Wavelet analysis

The sparsity of control variables makes it difficult to isolate the effect of complexity on growth. Fortunately, continuous wavelet tools (CWT) provide some relief. These tools have been applied in various fields of economics, such as political economy, health economics, and macroeconomics (Aguiar-Conraria et al., 2013, 2012; Luis Aguiar-Conraria et al., 2018; Luís Aguiar-Conraria et al., 2018; Antony and Klarl, 2020; Flor and Klarl, 2017; Klarl, 2016). To our knowledge, this paper is the first to use wavelet tools in the field of innovation economics. Unlike standard time series analysis methods, CWT can identify both short- and long-term relationships simultaneously, making it particularly suitable for analyzing the relationship between the complexity frontier and economic growth (Andersson, 2016).

CWT can account for relationships subject to multiple shocks (such as technological shifts) of different magnitudes and frequencies. It is possible that, for the same time period, a technological shock induces different causal lead-lag patterns between complexity and growth over various time lags (frequencies). This flexibility overcomes the limitations of traditional time-series methods and spectral analysis.

The data on the complexity frontier and economic growth exhibit oscillatory patterns and potential "*statistical noise.*" Wavelet analysis represents time series through a set of wave-like oscillations (wavelets), where all wavelets share the same functional form but differ in amplitude, length, and location on the time series. Some wavelet functions describe short-term patterns, while others describe long-term patterns. This differentiation, called "*frequency*" or "*scale*," allows for a comprehensive analysis. Repeating wavelets at different frequencies reduces the time series to its main patterns, simplifying the data and reducing potential noise.

When applied to two (or more) time series using the same wavelet function, it is possible to assess the similarity of the wavelets needed to describe each series. This is known as complex wavelet coherence, which measures the overlap or correlation between the wavelet representations of the time series. The degree of coherence is assessed by identifying potential Granger causal lead-lag relationships for different frequencies. This means that two time series can be Granger-causally related in their short-term or long-term patterns. Importantly, the researcher does not need to specify these patterns' precise characteristics, lengths, or temporal leads and lags (*phase-difference*); the analysis identifies and reports all possible configurations.

In the following analysis, the partial wavelet gain is of particular interest because it aligns with textbook econometrics: It can be interpreted as the coefficient in the multiple linear regression of the complexity frontier on economic growth and additional control variables. Three key pieces of information are obtained from the analysis: the partial phase difference, which provides insights into the direction of effects and lead-lag patterns; wavelet coherency, which indicates whether these patterns are statistically significant; and the partial wavelet gain, which quantifies the effect strength. For a more technical introduction to continuous wavelet analysis and a formal derivation of the partial wavelet gain, we refer the interested reader to Appendix A.1.

4. Results

4.1. The development of the complexity frontier

Based on the wavelet gain analysis of GDP per capita (GDP/c) and economic complexity, we assess models at various complexity thresholds (1%, 5%, 10%, and 50%) to determine the most suitable for interpretation. The results for the 5% threshold model are shown in Figure 7, with other specifications in Figure 13 in the Appendix.

The 5% threshold model demonstrates clear and distinct high coherence values at significant periods, particularly around 17 and 35 years, indicating strong co-movements between GDP/c and complexity. Additionally, the partial phase-difference curve for the 5% model exhibits smoother and more stable transitions over time compared to other thresholds, suggesting a more consistent and stable relationship. The gain values in this model are also higher and more stable than those at other thresholds, further reinforcing the robustness of the relationship between GDP/c and complexity at the 5% threshold. While the 1% and 10% scenarios show similar patterns to the 5% model, the 5% threshold outperforms them slightly in terms of stability and clarity. Conversely, the 50% threshold scenario performs worse, with more spread-out coherence values and lower, less stable gain values. These characteristics make the 5% model the most reliable and compelling for illustrating the dynamic interplay between economic growth and complexity. Moreover, it indicates that the relationship is most pronounced at this threshold. The poorer performance of the 50% threshold (median) strengthens the argument of Mewes & Broekel (2022) that an economy's technological capabilities relevant for economic growth are better approximated by the upper end of the complexity spectrum rather than the average complexity of the technologies it utilizes.

Consequently, for the presentation and discussion of the results, we focus on the complexity frontier with respect to the 5% percentile, i.e., the complexity value representing the average complexity of the 5% most complex technology usage instances.[7]

Figure 3 illustrates the development of the complexity frontier (CF5, red line) compared to other key variables (population, patents, GDP). It highlights that the complexity frontier grows faster than the number of patents, implying that the growth in technological complexity outpaces the quantity of innovation. Generally, the frontier increases (almost) monotonically, confirming the idea of increasing levels of technological complexity over time (Aunger, 2010; Broekel, 2019). It does feature a bump around the 1970s-80s, with the absolute level of complexity somewhat decreasing until a subsequent rise at the end of the 1980s. Overall, the frontier provides empirical support for the "*complexity thesis*" discussed in studies on cultural evolution (Vaesen and Houkes, 2017).

[7] Appendix A.2 features a more detailed presentation of the other thresholds' findings.

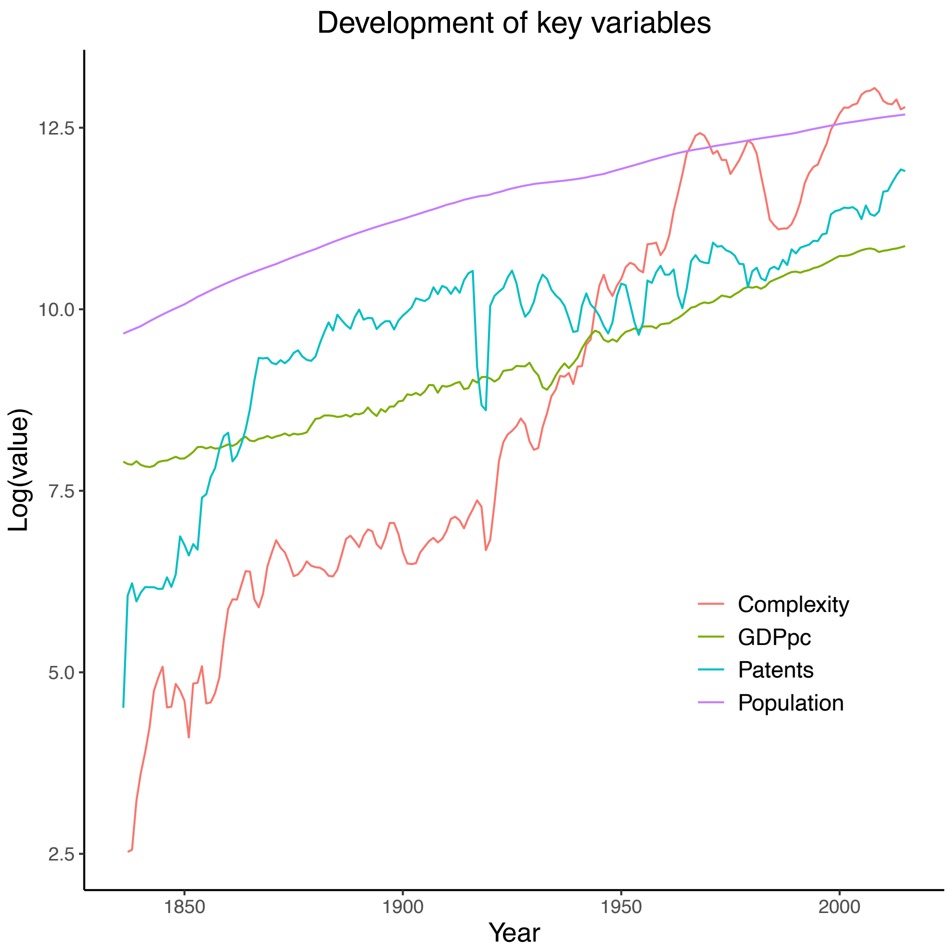

Figure 3: Development of key variables

During the 170 years, significant technological upheavals occurred, including the rise of electrics, chemistry, and computers. This raises the question of whether such changes are also evident in the evolution of technological complexity. The importance of each technology can be approximated by its relative contribution to the technology frontier. Since the frontier represents the 5% utilization instances of the most complex technologies in a year, we can calculate the share of instances each technology (4-digit CPC class) accounts for. These shares' values are smoothed over time using a loess regression with a span of 0.1 to highlight trends. To improve the visualization, they are transformed such that their sum corresponds to the complexity frontier value for that year (CF5). Particularly informative insights emerge when these technologies are aggregated into broader technological fields and sectors, revealing clearer trends that may not be apparent when examining individual technologies alone.

Following Balland et al. (2020) and Schmoch et al. (2003), Figure 4 visualizes the relative contribution of 10 aggregated sectors (proportionally presented by the colored ribbons), and Figure 5 shows the aggregation to 35 fields.[8] Both visualizations highlight the massive technological shifts that have taken place. At least three phases are visible in which single sectors dominate the

[8] One technology field is excluded from the analysis due to insufficient patent numbers. Visualizations without the smoothing of share values are presented in Appendix A.3 (see Figures 14 and 15).

frontier. Mechanical and civil engineering dominated in the first phase (until the 1920s). The second phase, from the 1920s to the 1990s, can be described as the chemical revolution, with technologies related to macromolecular chemistry and polymers exploding in the 1920s and subsequently dominating the frontier. Computer and telecommunication technologies initiated the third phase (in the 1990s), which shows semiconductor-related technologies (H01L) contributing disproportionately (compared to their overall patent shares) to the frontier until telecommunications (H04J) and digital communications (H04W) started to have higher shares from around the 2010s onward.

Looking at the aggregation of technologies into 35 technological fields (Figure 5) reveals a pronounced relationship between the evolution of the frontier and changes in the composition of contributing technologies. Early increases in complexity (1837-1900) were driven by diversified contributions from basic communication processes, electrical machinery, and audio-visual technology. A significant spike around 1950 corresponds with the emergence of computer technology, medical technology, and materials/metallurgy, highlighting the impact of these fields on overall complexity. The late 20th century saw another rise, driven by semiconductors, pharmaceuticals, and digital communication. In recent decades, high levels of complexity have been sustained by the emergence and growth of advanced technologies like semiconductors and IT methods.

If one is willing to accept macromolecular technologies as a representation of the oil, car, and aircraft sectors, one can find some resemblance of the idea of the so-called K-waves (Kondratieff cycles) in the shifting composition of the complexity frontier. Accordingly, the technical revolution from 1880 to 1920 was characterized by advances in chemistry, electrotechnical industry, and machinery. At least the latter's growing importance is also visible in the complexity frontier. Subsequently, between 1940-1970, the scientific-technical revolution fueled the rise of the oil-, synthetic materials-based sectors and motorization (macromolecular technologies) (Šmihula and Von, 2010). It was followed by the age of computerization (1973 onward) that overlaps with the ICT revolution in 1985-2000 and the subsequent growth of telecommunications, cybernetics, and informatics (Freeman and Louca, 2001; Šmihula and Von, 2010). The K-waves are very general, and their scientific basis and empirical evidence are intensely debated. However, the frontier cannot be seen as an empirical test for the validity of the K-waves argumentation, as the latter is not related to the concept of complexity. Consequently, we merely acknowledge certain correspondences between what this literature argues and the visual impressions of the complexity frontier.

The frontier also features a movement away from technologies filed under ‘Other Fields’ from 1925 onward and Mechanics (Other special machinery). Both just kept a marginal presence in the frontier after this time. However, the latter suddenly re-emerges with a noticeable contribution to the frontier in the 1980s and 1990s. This is driven by increased patenting associated with classes B29C[9] and B29K.[10] These are elementary classes for inventions related to composite materials such as Kevlar. This increase in these classes’ complexity mirrors the rise of modern composites invented in the 1960s, which found their way into a wide range of applications in the 1980s to 2000s, including the automobile, aircraft, construction, and military technologies (ŠERIFI et al.,

[9] Shaping or joining of plastics; Shaping of substances in plastic state, in general; After-treatment of the shaped products, e.g., repairing.

[10] Indexing scheme associated with subclasses B29B, B29C, or B29D, relating to moulding materials or to materials for reinforcements, fillers, or preformed parts, e.g., inserts.

2018). In recent years, several technologies expanded their contribution to the frontier noticeably, joining the dominating Digital Communications technologies. This includes the propulsion of electrically-propelled vehicles (B60L), laboratory apparatus (B01L), and the conjoint control of vehicles (B60W), which, accordingly, may have the potential to shape technological evolution in the near future.

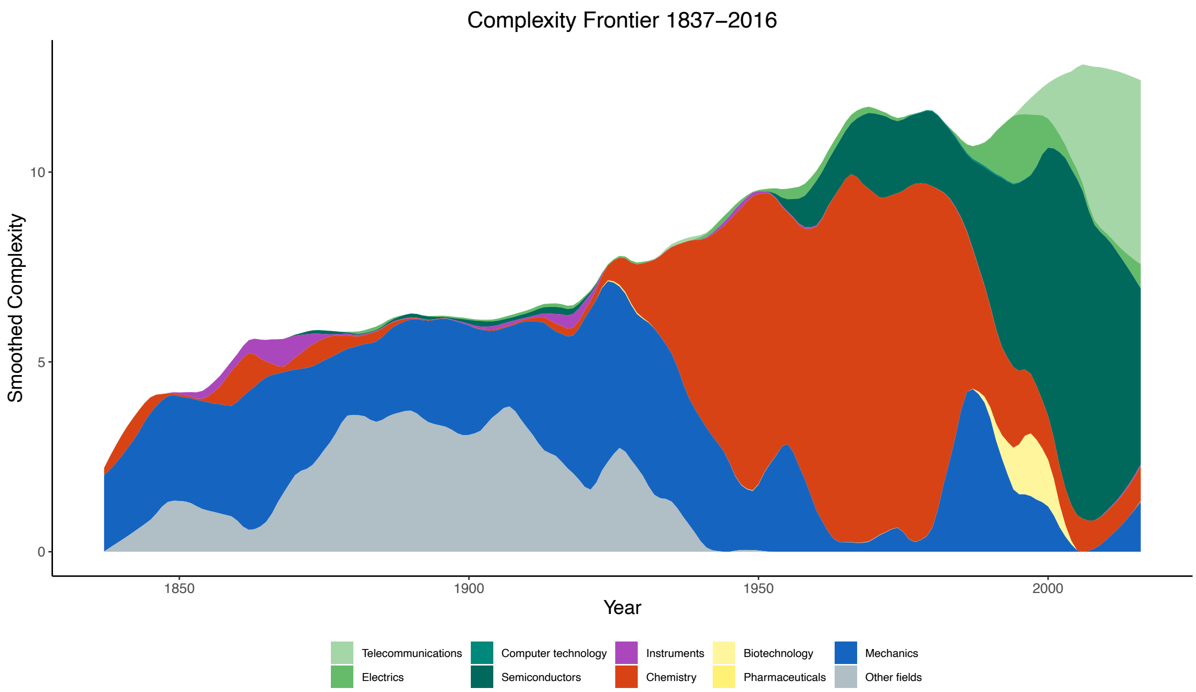

Figure 4: Complexity Frontier, sectorial composition

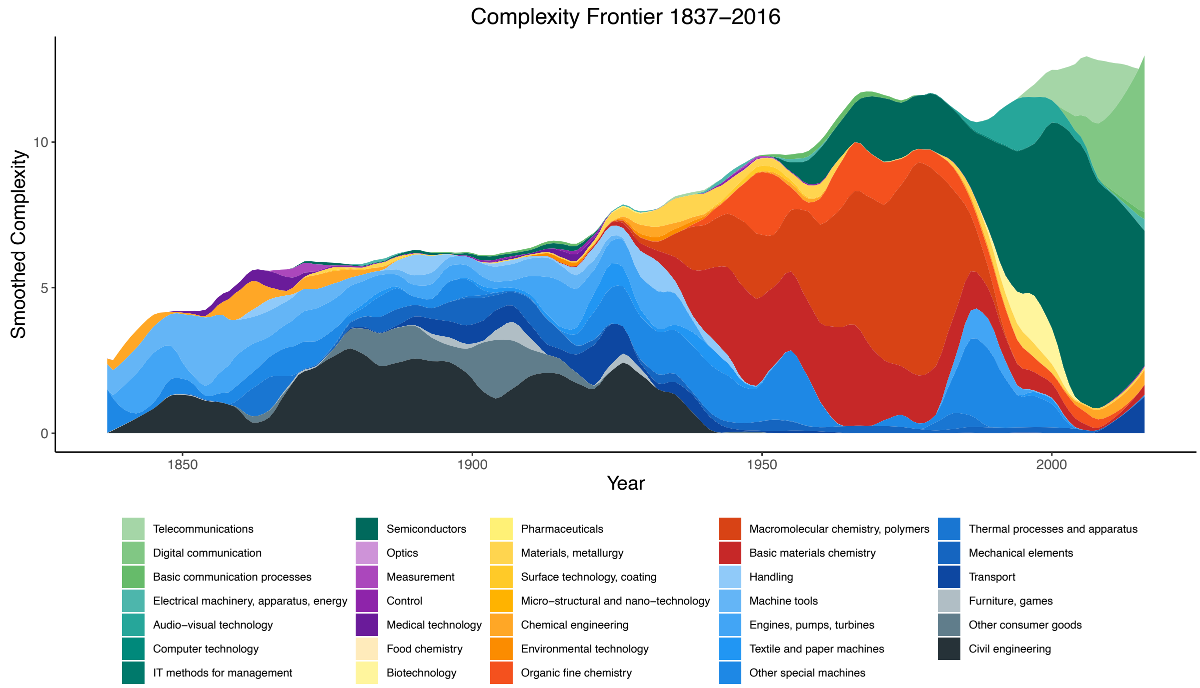

Figure 5: Complexity Frontier, field composition

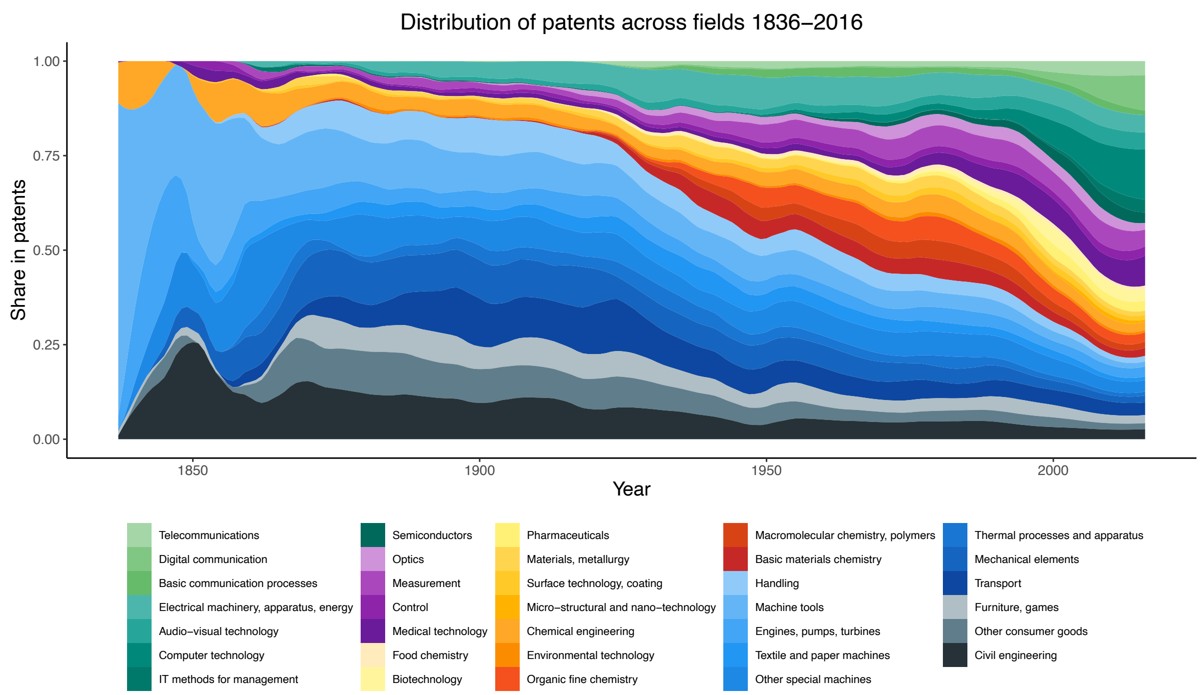

Figure 6: Distribution of patents across fields

This descriptive look at the frontier highlights its ability to magnify major technological shifts that are otherwise less (if at all) visible. For instance, the rise of macromolecular chemistry in the 1960s and 1970s and that of semiconductors does not become apparent when looking at the general distribution of technologies among all patent applications (see Figure 6). The next subsections will assess whether it also helps explain economic growth.

4.2. Complexity and Economic Growth

The presentation of the results of the wavelet-gain analysis diverges from the ordinary regression analysis tables. While all control variables are considered in the estimations, the results focus on insights into the core explanatory variable's (here, the *complexity frontier* - CF5) relationship with the dependent variable (GDP). The properties of this relationship are expressed in the three plots jointly shown in Figure 7. The heat map on the left indicates the significance level of the link between the capabilities in complex technologies and GDP growth, with the x-axis representing the year and the y-axis the length (in years) of the considered temporal variance of the two variables (also known as *frequency bands*). The dotted grey line is the so-called *cone of influence*, i.e., the period covered by our data for which we can reliably assess the statistical significance of the relationship. The areas outside the cone represent situations characterized by insufficient numbers of observations. The plot in the middle visualizes the *partial phase-difference*, which gives insights into the direction of impact: values (the blue line) within the interval -pi/2 and pi/2 indicate the complexity frontier impacting GDP growth, and values outside the interval suggest the impact going in the opposite direction. The last plot on the right-hand side represents the partial wavelet gain, which indicates the magnitude of the impact. For instance, a value of one means that if the complexity of the most complex technologies in which the economy has capabilities increases by one percent, it will translate into a one percent increase in GDP per capita. Given the richness of the information in these plots, we focus the results' discussion on two time periods for which a statistically significant ($p<0.05$) relationship is identified.

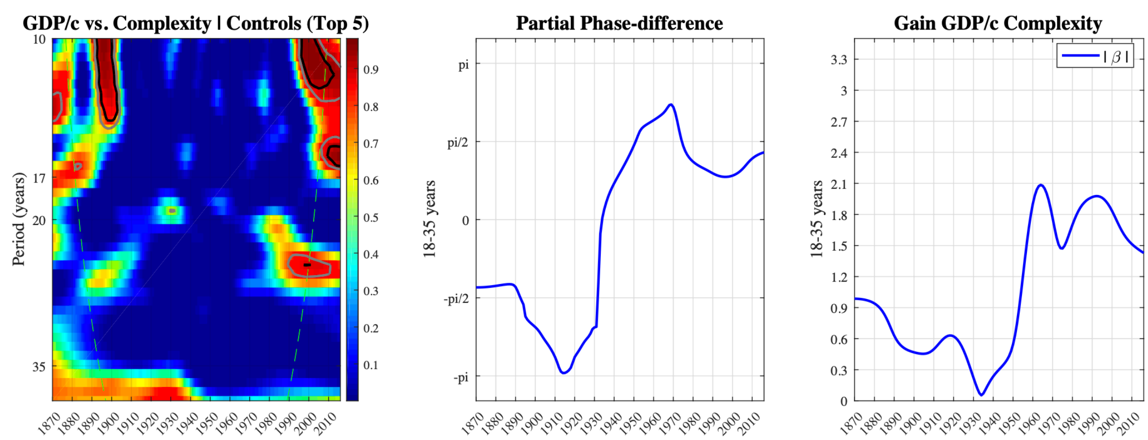

Figure 7: Complexity frontier (5% most complex) and GDP/c growth

4.2.1. Developments between 1890-1907

We find a significant (5% level of significance) relationship between the complexity frontier and economic growth between 1890 and 1907, considering a 10-15-year temporal variance. The relationship is significantly negative during this time, whereby GDP/c leads and Granger-causing the complexity frontier values. This is indicated by the partial phase difference being in the interval -pi/2 and -pi in the middle plot of Figure 7. The wavelet gain (right panel) indicates that an increase of GDP/c by 1% led to a decrease in the frontier value by 0.35% during this time. This period falls within the "*technical revolution*" from 1880 to 1920 (Šmihula and Von, 2010). At the end of the 19th century, the growth of the US changed from being based on expanding capital, resources, and labor to knowledge-based growth (Abramovitz and David, 1999; Mowery, 2010). Driven by the desire to expand economic output further, resource-extraction sectors led investments in R&D activities in related fields (chemistry, machine tools, electricity), in which the US soon became a global force at the beginning of the 20th century (Mowery, 2010; Nelson and Wright, 1992). This expansion materialized in absolute patent numbers doubling during this time (see Figure 9). However, our analysis shows that it did not lead to substantial shifts and changes in the complexity frontier (see Figure 7 and Figure 8), which remained dominated by capabilities in the same technologies (civil engineering, other consumer goods, and mechanics). The frontier also didn't shift upwards, implying that these technologies did not drive an upgrading of the economy's capabilities in terms of inventing and utilizing more complex technologies (see Figure 7 and Figure 8). Therefore, the negative relationship between GDP/c growth and the complexity frontier between 1890 and 1907 appears to be a consequence of the positive contribution of resource-extraction sectors to economic growth and their investments into R&D in (comparatively) simpler technologies. This development coincides partially with the "*productivity pause*" between 1890 and 1914, during which low productivity gains didn't stimulate economic growth (David and Wright, 1999). According to our study, this phenomenon is evident in and seems to be related to, a sideways development (or even decrease) of the complexity frontier.

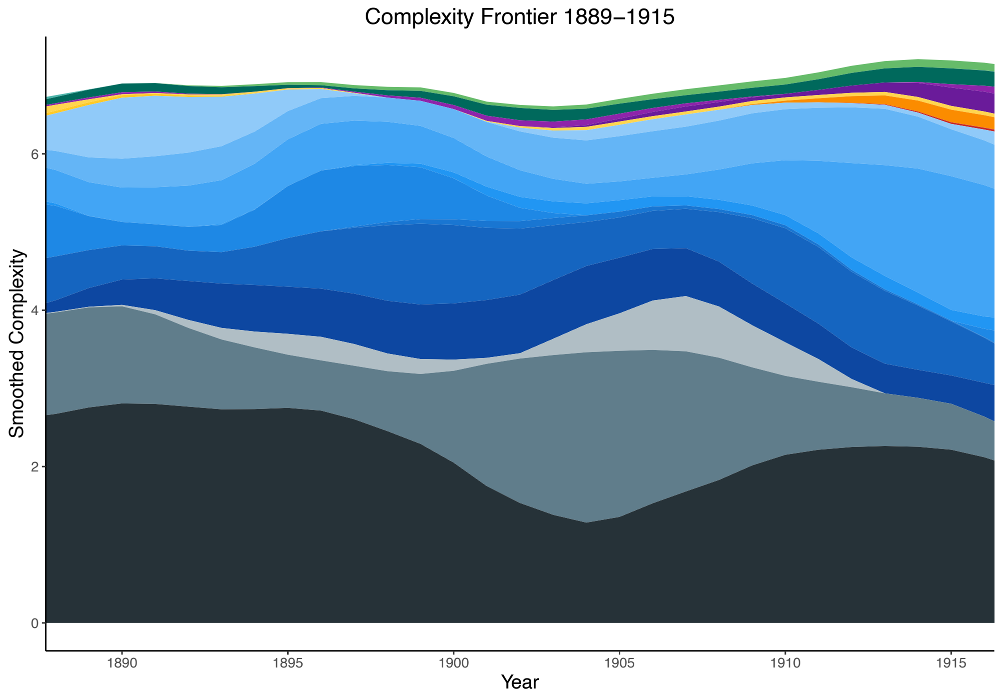

Figure 8: Complexity Frontier 1889-1915

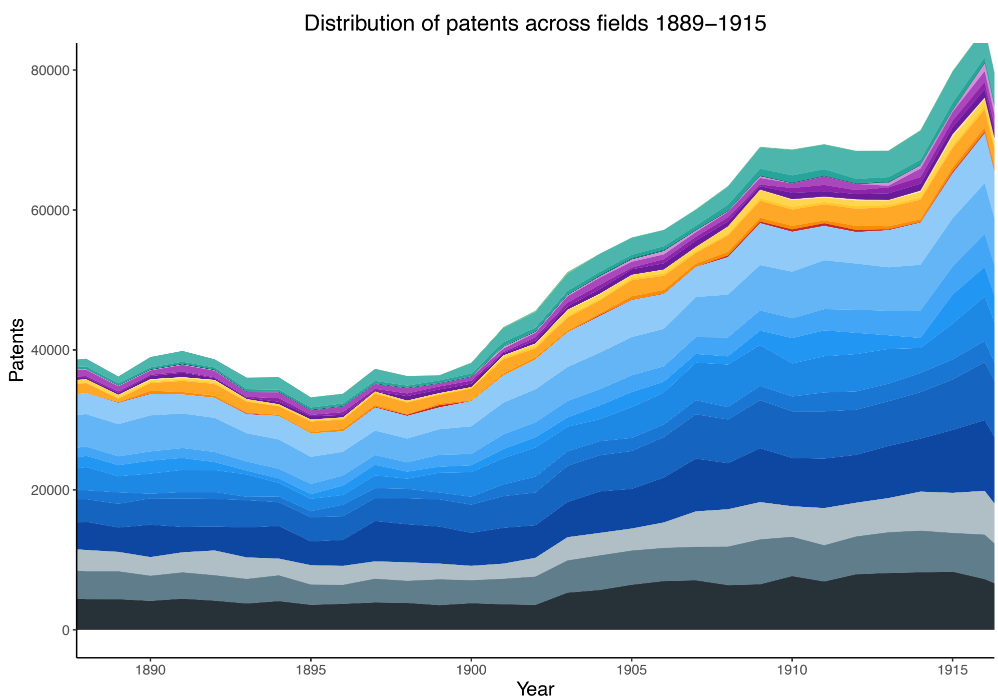

Figure 9: Patent numbers 1889-1915

4.2.2. 1990 onward

The second period for which we observe a significant (5% level of significance) Granger-causal relationship between the complexity frontier and GDP growth is between 1990 and 2011. Increases in the complexity frontier (CF5) significantly contributed to economic growth during this time. While the relationship seems to have continued after 2011, we cannot reliably assess its significance after this year because the method requires a time window of at least five years, and our data ends in 2016. During this period, the rise of ICT technologies caused a sharp increase in patent growth (see Figure 11). However, the growing importance of these technologies is more visible when looking at the composition of the complexity frontier. Here, semiconductors downright squeezed out chemistry from the 1970s onward (Figure 10). According to the wavelet-gain analysis, this improvement in the economy's capabilities to handle more complex technologies translated into considerable economic growth (see wavelet gain plot in Figure 7). More precisely, an increase in the complexity frontier by 1%, which roughly corresponds to what occurred between 1990 and 2011, caused GDP to grow by 1.8%. This impact of expanding capabilities in complex ICT technologies on economic growth aligns with other studies that identified ICT to substantially contribute to productivity growth during this period (Brill et al., 2018; Niebel, 2018; Oliner and Sichel, 2000). Yet, the impact is substantial and significantly larger in magnitude than what is reported for the regional level in Europe: Mewes and Broekel (2022) found that an increase in the capacity to handle technologies of 10% greater complexity is associated with an approximate 0.45% uptick in GDP growth. A potential explanation for this difference is that the US outgrew Europe economically during this period. Much of this is attributed to the comparatively greater presence and growth of the ICT sector and its technologies in the US (van Ark et al., 2003).

In the wavelet analysis, the impact of the capabilities in complex technologies on growth is shown in both variables' temporal variations overlapping in multiple scenarios. It is significant considering the 10 to 15 years temporal variance and the one over 15 to 17 years. Consequently, improved technological capabilities require some time to manifest into economic growth. Potentially, this is because the relevant technologies (ICT and related technologies) are general-purpose technologies (Beaudry et al., 2015). Such technologies shape productivity development in a wide range of sectors, which implies that it takes a considerable time period before they diffuse in the economy, and their application has macroeconomic effects.

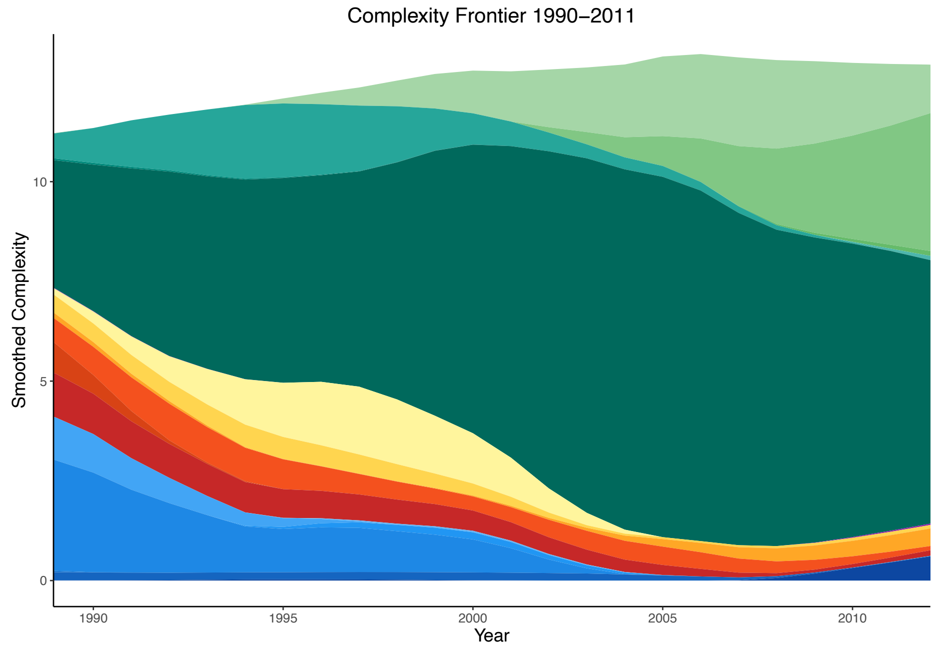

Figure 10: Complexity Frontier 1989-2011

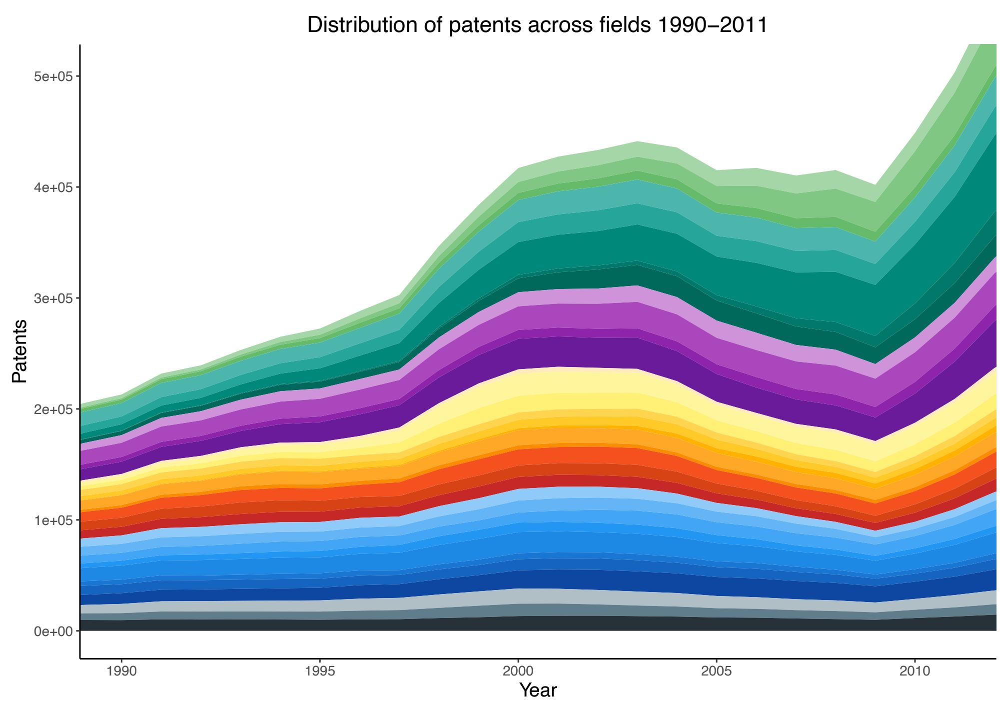

Figure 11: Patent numbers 1989-2011

4.2.3. The intensity of inventive activities and economic growth

Including the number of patents in the estimations controls for changes in R&D efforts, inventive success, and intellectual property protection strategies over time. It is also crucial to isolate the effect of the capabilities in complex technologies, as the measure of structural diversity is positively correlated with the number of patents. However, this variable may also reflect the general intensity of inventive activities in a given year. Put more loosely, when technological complexity is seen as a more qualitative dimension of inventive activities, absolute patent numbers rather represent their quantity (Mewes and Broekel, 2022). To validate that both variables truly pick up independent effects, we re-estimate the models with the values of the complexity frontier and the total number of patents switching their places. That is, we test for the relationship between GDP growth and the development of patent numbers over time with the complexity frontier as a control variable. Figure 12 visualizes the results. Interestingly, we observe a significant relationship between economic growth and patent numbers for the time before 1910 and for the period after 1990.

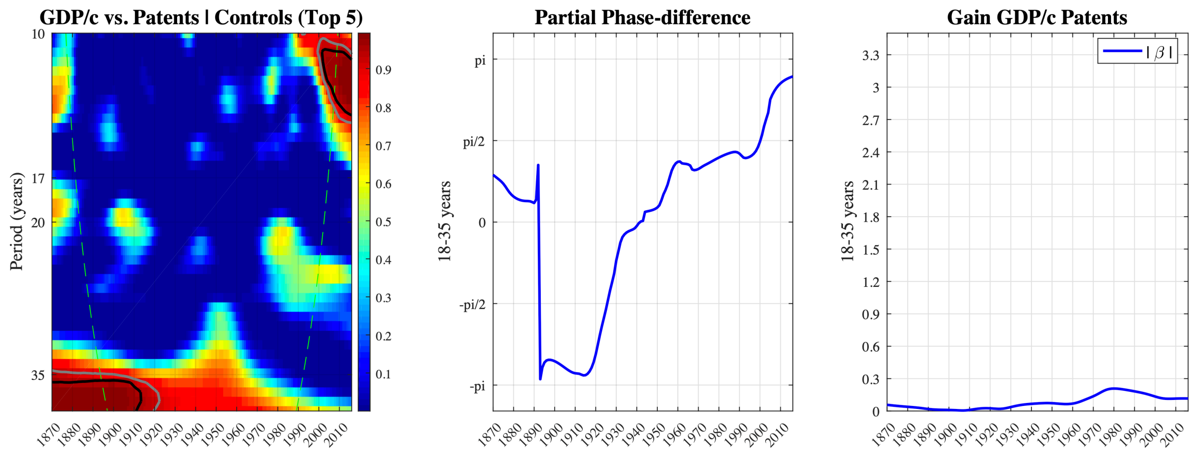

Figure 12: Total number of patents and GDP/c growth

At first glance, this seems similar to what is observed for the complexity frontier. However, there are two substantial differences. Firstly, the growth of patents is Granger-causally driven by changes in GDP in both periods (before 1910 and after 2000). It is only briefly reversed between 1990-2000. In contrast, the capabilities in complex technologies remained an economic facilitator in all years after 1990. Secondly, even in the period where the influence of patents and the complexity frontier on economic growth is simultaneously significant (1990-2000), the wavelet gain (strength of influence) is much lower for patent numbers (ca. 0.3%) than for complexity (ca. 1.8%).

Consequently, the two dimensions (number of patents and capabilities in complex technologies) represent distinct influences, implying that the relevance of the technological capabilities for economic growth is not only a result of increased patenting.

5. Discussion and conclusion

Innovation and technological progress are unquestioned in their importance for economic growth. Yet, quantitative empirical evidence that covers a long time and considers the heterogeneous nature of technologies and their variance in impacting economic growth is scarce. Motivated by this scarcity, the present work utilized patent data and Wavelet-Gain Analyses to investigate the relationship between advancements in technological capabilities and economic growth for the US economy from 1840 to 2016. The study especially considered changes in the economy's capability to invent and utilize the most complex technologies because these are seen as decisive for adding new economic growth opportunities (Antonelli et al., 2020; Hidalgo et al., 2009; Mewes and Broekel, 2022). Our empirical investigation shows that economic development and the economy's ability to expand its competencies in more complex technologies, i.e., pushing the complexity frontier, are not related for the longest time. Two exceptions are the period 1890-1907 and the time after 1990. Contrary to our expectations, between 1890 and 1907, economic growth enabled an expansion of the complexity frontier, i.e., economic expansion drove the capabilities to invent and utilize complex technologies. Only in the years after 1990 is our primary hypothesis (**H1**) confirmed. From this time onward, advancing capabilities in complex technologies translated into economic growth. Accordingly, since 1990, the US, and potentially other advanced economies, have entered the age of complexity-driven growth. Thereby, our study confirms existing insights into capabilities in complex technologies influencing economic growth for Europe from 2000 to 2014 (Mewes and Broekel, 2022) and for China between 1991 and 2015 (Li and Rigby, 2022). We add a macroeconomic study covering a much longer (170 years) period to this existing evidence at the regional level.

Yet, our study goes beyond existing empirical insights by providing empirical support for the time-variant nature of the relationship between technological capabilities and economic growth (hypothesis **H2a**). Advancing technological capabilities has not always been the strongest direct driver of economic growth in the history of the USA. In particular, during the 19th and the beginning of the 20th century, much of its growth was caused by the expansion of its population, natural resources, infrastructure, and mass production (Abramovitz and David, 1999; Mowery, 2010). This doesn't mean that technological capabilities and capabilities in the most complex technologies didn't matter. They were just not the decisive forces and potentially impacted growth by contributing to the effects of the other growth factors. Consequently, future research needs to explore the potential indirect contributions of these capabilities to economic development, such as economic transformation and diversification (Gala et al., 2018; Hartmann, 2018; Hidalgo et al., 2009; Pugliese et al., 2017).

Our study also supports our last hypothesis, **H2b**, implying that the positive link between the capabilities to advance technological complexity and economic growth is more pronounced in recent periods. While our data only allows inferring about this relationship until about 2010 statistically, it provides solid indications for it to continue afterward as well. The positive effect of the capabilities on growth coincides with the ICT revolution. These technologies are widely acknowledged as being highly complex as they "are highly cumulative and interactive, requiring a great deal of interoperability between components made by different firms, which has increased the importance of standards, collaboration among firms, and network effects in adoption" (Hall and Rosenberg, 2010, p. 6). Inventing and utilizing these highly complex technologies pushed the complexity frontier, which our study empirically documents. Simultaneously, the effect of capabilities in complex technologies on growth became statistically significant. It empirically confirms the ICT technologies' contribution to economic growth, which Robert Solow famously questioned in the 1980s (Solow, 1987). Yet, our findings provide some support for this view, as its contribution was empirically not visible at the time. From the 1990s onward, its impact is statistically significant, implying that our findings add further evidence to the rise of ICT technologies contributing to economic growth even though it might still be less than that of previous technological revolutions (Gordon, 2018; Pilat, 2005). Yet, this strong influence may be specific to the USA, where a much larger share of the largest R&D performing and fastest growing firms were founded during this time than in European countries (Hall and Rosenberg, 2010). Given the importance of standardization for the applicability of ICT technologies, this finding links to their documented contribution to economic growth (Blind et al., 2022; Blind and Jungmittag, 2008). It is beyond the scope of this paper to expand on this link, which we have to leave to future studies.

In addition to being limited to the case of a single country, the USA, our study has further limitations that need to be pointed out. Most importantly, we rely on patent data to approximate inventive activities, technological focus, and the capabilities to invent and utilize complex technologies. Our findings are as reliable as these approximations reflect reality. While patent data is known to be subject to multiple biases and clearly gives only a partial representation of these dimensions (Griliches, 1990), so far, there are no alternative data sources that cover such a long period and that provide such a level of detail on technological developments. Consequently, we must leave it to future research to compare these insights with alternative approaches to capture economic development and technological capabilities. Recent works assessing economic and skill complexity are promising research starting points (e.g., Hidalgo et al., 2009; Waters and Shutters, 2022). In addition, the approximation of technologies' complexity is exclusively based on US patents, which may be an inadequate representation of what constitutes the most complex technologies when the USA was not the technology leader.

Our investigation is also limited with respect to considering confounding factors for which long-term data is scarce. While we are confident that the employed partial wavelet-gain analysis can still identify the causal relationship, we cannot completely rule out that some confounding factors may alter the empirical findings. Most importantly, we lack information on the distribution and magnitude of R&D investments and on the diffusion of technologies across the US economy. This prevents exploring the degree to which increasing complexity contributes to the decreasing R&D productivity observed by Strumsky et al. (2010) and Bloom et al. (2017). Theoretically, growing technological complexity makes research increasingly costly (Kim, 2015; Strumsky et al., 2010), induces longer development times (Griffin, 1997), and implies greater chances of failure (Singh, 1997) than doing the same at a lower level of complexity. However, this has yet to be empirically confirmed at the level of an economy over an extended period because of the limited available data.

Consequently, while it seems plausible that growing complexity reduces R&D productivity, motivating the massively growing R&D investments to ensure a relatively constant rate of progress, it remains speculative currently. However, our findings suggest that the net effect, complexity-induced costs vs. growth-enhancing force, is not negative. That is, we only find capabilities in complex technologies to facilitate economic growth at a time when it has reached the highest level (so far observed) and when the costs it induces on R&D should be at a peak as well.

Nevertheless, it clearly points to another limitation of the present study. We reduced the complexity frontier and its development into a single indicator reflecting the average complexity of the most complex technological activities. However, the frontier contains much more information than this. Most importantly, it highlights the changing order of technologies in terms of their levels of complexity over time. In the late 1930s, chemistry became the dominant technology in the frontier before it was replaced by semiconductors in the late 1980s. This change meant more than just a new spurt in complexity growth. For instance, it was accompanied by a shift in stock market valuations. In 1967, eight of the 20 largest companies (by market value) belonged to the Chemical or Oil & Gas sector. Only three of the top 20 were from the Tech sector. In 2017, eight companies were from Tech or Telecom, and only two were from Oil & Gas (Kauflin, 2017). This shift also coincides with a change in the geography of innovation, which became most visible in the rise of the US West Coast as a new center of R&D and innovation (Saxenian, 1994). Hence, future research needs to widen the perspective and disentangle these developments and their relationship with technological complexity in more detail.

Notwithstanding these limitations, our study suggests that technological complexity has the potential to become a central pillar in contemporary technology and innovation policies. While complexity is only indirectly considered in it, the EU's smart-specialization strategy represents a forward step (Balland et al., 2018; Deegan et al., 2021). Our findings indicate that the relationship between capabilities in complex technologies and economic growth becomes evident over a medium to long-term horizon of ten to seventeen years. Consequently, technology policies that emphasize the development of complex technologies must be designed with a long-term perspective to capture their growth potential fully.

Adapting to technological change is critical for countries looking to harness the benefits of capabilities in technological complexity. Policies should be agile and forward-looking, anticipating changes in the technology landscape and responding proactively. For example, regulatory frameworks should evolve to address the rapid advancements in complex technological domains, ensuring that innovation is not stifled by outdated regulations. Our paper illustrates a way of identifying such potent complex technologies. A recent analysis by Nast et al. (2024) of the USA's technological development over the past decades shows that contemporary governmental support schemes promote innovation in more complex technologies. Whether this is economically beneficial and whether similar patterns can be observed in other countries remains to be explored. However, the period during which Nast et al. (2024) identify governmental support pushing technological complexity coincides with the period in which our study shows that complexity drove economic growth. This alignment suggests that government support can be crucial in fostering technological advancements that contribute to economic growth.

Regarding industrial policy, targeted support for industries operating within complex technological fields is essential. This could involve strategic R&D investments in sectors with high growth potential, fostering the development of industry clusters that promote knowledge exchange and innovation. Incentives such as subsidies, tax breaks, or direct investments can be instrumental in cultivating industries that leverage complex technologies for economic growth.

To achieve sustainable growth, policymakers must also consider the dynamic nature of technological complexity. As industries evolve, so too must the policies that support them. This includes the development of a skilled workforce capable of advancing these technologies, as well as infrastructure that facilitates the growth of high-tech industries. By doing so, countries can ensure that they not only keep pace with technological advancements but also lead in creating and applying new and complex technologies.

Ultimately, the successful integration of complex technologies into the economic fabric of a country relies on the coordinated efforts of educational institutions, industry, and government. Countries can hope to achieve long-term economic prosperity only through a sustained commitment to understanding and applying the principles of technological complexity.

However, considering the limited insights into the development of technological complexity over time and space, several crucial questions need to be answered first. Is it a sustainable policy to push the complexity further, which may generate more economic growth while making additional advancements in the future more difficult and costly? Is the research and development system designed to deal with even more complex technologies? Are complex and simple technologies complements or substitutes?

Lastly, in the context of explaining economic growth, our study advocates the position of economic complexity (Hidalgo et al. 2009; Sweet and Eterovic, 2009; Balland et al., 2009) as a pivotal indicator from a capability standpoint, encapsulating the multidimensional capabilities required to navigate complexity. While our research emphasizes the technological aspect, we recognize the significance of other dimensions, notably skill and institutional complexity. We advocate for future research to elucidate the distinct impacts of these various complexity dimensions on economic growth, further enriching our understanding of the drivers behind economic advancement.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author(s) used OpenAI's ChatGPT to improve the writing. After using this tool, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

References

AAAS, 2022. Federal R&D Budget Dashboard.

Abramovitz, M., David, P.A., 1999. American macroeconomic growth in the era of knowledge-based progress: The long-run perspective.

Aghion, P., David, P.A., Foray, D., 2009. Science, technology and innovation for economic growth: Linking policy research and practice in `STIG Systems'. Res. Policy 38, 681–693.

Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60, 323–351.

Aguiar-Conraria, L., Magalhães, P.C., Soares, M.J., 2013. The nationalization of electoral cycles in the United States: A wavelet analysis. Public Choice 156, 387–408. https://doi.org/10.1007/s11127-012-0052-8

Aguiar-Conraria, L., Magalhães, P.C., Soares, M.J., 2012. Cycles in Politics: Wavelet Analysis of Political Time Series. Am. J. Pol. Sci. 56, 500–518. https://doi.org/10.1111/j.1540-5907.2011.00566.x

Aguiar-Conraria, Luis, Martins, M.M.F., Soares, M.J., 2018. Estimating the Taylor rule in the time-frequency domain. J. Macroecon. 57, 122–137. https://doi.org/10.1016/j.jmacro.2018.05.008
Aguiar-Conraria, L., Soares, M.J., 2014. The continuous wavelet transform: Moving beyond uni- and bivariate analysis. J. Econ. Surv. 28, 344–375. https://doi.org/10.1111/joes.12012

Aguiar-Conraria, Luís, Soares, M.J., Sousa, R., 2018. California's carbon market and energy prices: A wavelet analysis. Philos. Trans. R. Soc. A Math. Phys. Eng. Sci. 376. https://doi.org/10.1098/rsta.2017.0256

Albeaik, S., Kaltenberg, M., Alsaleh, M., Hidalgo, C.A., 2017. Improving the Economic Complexity Index. Arixv Work. Pap. arXiv:1707, 1–21.

Andersson, F., 2016. Identifying and modelling cycles and long waves in economic time series, in: Identifying and Modelling Cycles and Long Waves in Economic Time Series. Routledge, London.

Antonelli, C., 1995. The Economics of Localized Technological Change and Industrial Dynamics. Kluwer, Dordrecht.

Antonelli, C., Crespi, F., Quatraro, F., 2020. Knowledge complexity and the mechanisms of knowledge generation and exploitation: The European evidence. Res. Policy 104081. https://doi.org/10.1016/j.respol.2020.104081

Antony, J., Klarl, T., 2020. Estimating the income inequality-health relationship for the United States between 1941 and 2015: Will the relevant frequencies please stand up? J. Econ. Ageing 17, 100275. https://doi.org/10.1016/j.jeoa.2020.100275

Arthur, W.B., 2021. Foundations of complexity economics. Nat. Rev. Phys. 3, 136–145. https://doi.org/10.1038/s42254-020-00273-3

Aunger, R., 2010. Types of technology. Technol. Forecast. Soc. Change 77, 762–782.

Balland, P.-A., Broekel, T., Diodato, D., Giuliani, E., Hausmann, R., O'Clery, N., Rigby, D., 2022. The new paradigm of economic complexity. Res. Policy 51, 104450. https://doi.org/10.1016/j.respol.2021.104450

Balland, P.-A., Rigby, D., 2017. The Geography of Complex Knowledge. Econ. Geogr. 93, 1–23. https://doi.org/10.1080/00130095.2016.1205947

Balland, P.-A.A., Boschma, R., Crespo, J., Rigby, D.L., 2018. Smart specialization policy in the European Union: relatedness, knowledge complexity and regional diversification. Reg. Stud. 53, 1252–1268. https://doi.org/10.1080/00343404.2018.1437900

Balland, P.A., Jara-Figueroa, C., Petralia, S.G., Steijn, M.P.A., Rigby, D.L., Hidalgo, C.A., 2020. Complex economic activities concentrate in large cities. Nat. Hum. Behav. 4, 248–254. https://doi.org/10.1038/s41562-019-0803-3

Beaudry, P., Green, D.A., Sand, B.M., 2015. The Great Reversal in the Demand for Skill and Cognitive Tasks. J. Labor Econ. 34, S199–S247. https://doi.org/10.1086/682347

Blind, K., Jungmittag, A., 2008. The impact of patents and standards on macroeconomic growth: a panel approach covering four countries and 12 sectors. J. Product. Anal. 29, 51–60. https://doi.org/10.1007/s11123-007-0060-8

Blind, K., Ramel, F., Rochell, C., 2022. The influence of standards and patents on long-term economic growth. J. Technol. Transf. 47, 979–999. https://doi.org/10.1007/s10961-021-09864-3

Bloom, N., Jones, C., Van Reenen, J., Webb, M., 2017. Are Ideas Getting Harder to Find? NBER Work. Pap. 23782. https://doi.org/10.3386/w23782

Bolt, J., Inklaar, R., Jong, H. de, Zanden, J.L. van, 2018. Rebasing 'Maddison': new income comparisons and the shape of long-run economic development. Maddison Proj. Work. Pap. 10.

Bresnahan, T.F., Trajtenberg, M., 1995. General purpose technologies 'Engines of growth'? J. Econom. 65, 83–108. https://doi.org/10.1016/0304-4076(94)01598-T

Brill, M., Chansky, B., Kim, J., 2018. Multifactor productivity slowdown in U.S. manufacturing. Mon. Labor Rev. 2018, 1–15. https://doi.org/10.21916/mlr.2018.19

Broekel, T., 2019. Using structural diversity to measure the complexity of technologies. PLoS One 14, 1–27. https://doi.org/10.1371/journal.pone.0216856

Broekel, T., Knuepling, L., Mewes, L., 2023. Boosting, sorting and complexity—urban scaling of innovation around the world. J. Econ. Geogr. 23, 979–1016. https://doi.org/10.1093/jeg/lbad006

Buarque, B.S., Davies, R.B., Hynes, R.M., Kogler, D.F., 2020. OK Computer: the creation and integration of AI in Europe. Cambridge J. Reg. Econ. Soc. https://doi.org/10.1093/cjres/rsz023

Carlino, G.A., Chatterjee, S., Hunt, R.M., 2007. Urban density and the rate of invention. J. Urban Econ. 61, 389–419. https://doi.org/10.1016/j.jue.2006.08.003

Cazelles, B., Chavez, M., Berteaux, D., Ménard, F., Vik, J.O., Jenouvrier, S., Stenseth, N.C., 2008. Wavelet analysis of ecological time series. Oecologia 156, 287–304. https://doi.org/10.1007/s00442-008-0993-2

Chávez, J.C., Mosqueda, M.T., Gómez-Zaldívar, M., 2017. Economic complexity and regional growth performance: Evidence from the Mexican economy. Rev. Reg. Stud. 47, 201–219. https://doi.org/10.52324/001c.8023

Cohen, W.M., 2010. Fifty Years of Empirical Studies of Innovative Activity and Performance, in: Handbook of the Economics of Innovation. Springer, pp. 129–213. https://doi.org/10.1016/S0169-7218(10)01004-X

Cowan, R., David, P.A., Foray, D., 2000. The Explicit Economics of Knowledge Codification and Tacitness. Ind. Corp. Chang. 9, 211–253.

Crespo, J., Balland, P.-A., Boschma, R.A., Rigby, D., 2017. Regional Diversification Opportunities and Smart Specialization Strategies. EU-Directorate-General Res. Innov.

David, P., Wright, G., 1999. Early Twentieth Century Productivity Growth Dynamics: An Inquiry into the Economic History of Our Ignorance | Oxford Economic and Social History Working Papers | Working Papers. Discuss. Pap. Econ. Soc. Hist.

Deegan, J., Broekel, T., Fitjar, R.D., 2021. Searching through the Haystack The relatedness and complexity of priorities in smart specialisation strategies. Pap. Evol. Econ. Geogr. 21.23, 1–24. https://doi.org/10.1080/00130095.2021.1967739

Dosi, G., 1988. The nature of the innovative process, in: Dosi, G., Freeman, C., Nelson, R., Silverberg, G., Soete, L. (Eds.), Technical Change and Economic Theory. Pinter, London, pp. 221–238.

Emmert-Streib, F., Dehmer, M., 2012. Exploring statistical and population aspects of network complexity. PLoS One 7.

Evangelista, R., Meliciani, V., Vezzani, A., 2018. Specialisation in key enabling technologies and regional growth in Europe. Econ. Innov. New Technol. 27, 273–289. https://doi.org/10.1080/10438599.2017.1338392

Fleming, L., Sorenson, O., 2001. Technology as a complex adaptive system: evidence from patent data. Res. Policy 30, 1019–1039. https://doi.org/10.1016/S0048-7333(00)00135-9

Flor, M.A., Klarl, T., 2017. On the cyclicity of regional house prices: New evidence for U.S. metropolitan statistical areas. J. Econ. Dyn. Control 77, 134–156. https://doi.org/10.1016/j.jedc.2017.02.001

Freeman, C., Louca, F., 2001. As Time Goes By: From the Industrial Revolutions to the Information Revolution. Oxford University Press Inc., New York.

Gala, P., Rocha, I., Magacho, G., 2018. The structuralist revenge: economic complexity as an important dimension to evaluate growth and development. Brazilian J. Polit. Econ. 38, 219–236. https://doi.org/10.1590/0101-31572018v38n02a01

Galbraith, M.W., 1990. The nature of community and adult education. New Dir. Adult Contin. Educ. 1990, 3–11. https://doi.org/10.1002/ace.36719904703

Galende, J., 2006. Analysis of technological innovation from business economics and management. Technovation 26, 300–311.

Gambardella, A., Heaton, S., Novelli, E., Teece, D.J., 2021. Profiting from Enabling Technologies? Strateg. Sci. 6, 75–90. https://doi.org/10.1287/stsc.2020.0119

Glaeser, E.L., 2011. Triumph of the city: how our greatest invention makes us richer, smarter, greener, healthier, and happier. Penguin Press, New York.

Gordon, R.J., 2018. Declining American economic growth despite ongoing innovation. Explor. Econ. Hist. 69, 1–12. https://doi.org/10.1016/j.eeh.2018.03.002

Gordon, R.J., 2016. The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War, 1st ed. Princeton University Press.

Griffin, A., 1997. The Effect of Project and Process Characteristics on Product Development Cycle Time. J. Mark. Res. 34, 24–35.

Griliches, Z., 1990. Patent statistics as economic indicators: A survey. J. Econ. Lit. 28, 1661–1701.

Grimaldi, R., Torrisi, S., 2001. Codified-tacit and general-specific knowledge in the division of labour among firms - A study of the software industry. Res. Policy 30, 1425–1442.

Hall, B.H., Rosenberg, N., 2010. Introduction to the Handbook, in: Handbook in Economics. pp. 3–9. https://doi.org/10.1016/S0169-7218(10)01001-4

Hartmann, D., 2018. Economic complexity and human development: How economic diversification and social networks affect human agency and welfare, Economic Complexity and Human Development: How Economic Diversification and Social Networks Affect Human Agency and Welfare. https://doi.org/10.4324/9780203722084

Hausmann, R., Hidalgo, C.A., Bustos, S., Coscia, M., Chung, S., Jiminez, J., Simoes, A., Yildirim, M.A., 2013. The Atlas of economic complexity: mapping paths to prosperity. Center for International Development, Harvard University, Cambridge.

Herrendorf, B., Rogerson, R., Valentinyi, Á., 2014. Growth and Structural Transformation, Handbook of Economic Growth. Elsevier B.V. https://doi.org/10.1016/B978-0-444-53540-5.00006-9

Hidalgo, C.A., 2023. The policy implications of economic complexity. Res. Policy 52, 104863. https://doi.org/10.1016/j.respol.2023.104863

Hidalgo, C.A., 2015. Why Information Grows: The Evolution of Order, from Atoms to Economies. Basic Books, New York.

Hidalgo, C.A., Hausmann, R., Hidalgo, A., Hausmann, R., 2009. The building blocks of economic complexity. Proc. Natl. Acad. Sci. U. S. A. 106, 10570–10575. https://doi.org/10.1073/pnas.0900943106

Howitt, P., 1999. Steady Endogenous Growth with Population and R . & D . Inputs Growing. J. Polit. Econ. 107, 715–730.

Hudgins, L., Friehe, C.A., Mayer, M.E., 1993. Wavelet transforms and atmopsheric turbulence. Phys. Rev. Lett. 71, 3279–3282. https://doi.org/10.1103/PhysRevLett.71.3279

Iscan, T., 2010. How much can Engel's Law and Baumol's disease explain the rise of service employment in the United States? B.E. J. Macroecon. 10. https://doi.org/10.2202/1935-1690.2001

Katkalo, V.S., Pitelis, C.N., Teece, D.J., 2010. Introduction: On the nature and scope of dynamic capabilities. Ind Corp Chang. 19, 1175–1186.

Kauflin, J., 2017. America's Top 50 Companies 1917-2017. Forbes.

Kerr, W.R., 2009. Breakthrough Inventions and Migrating Clusters of Innovation.

Khan, Z., 2006. The Democratization of Invention: Patents and Copyrights in American Economic Development, 1790–1920. Cambridge University Press, New York. https://doi.org/10.1162/jinh.2007.38.1.136

Kim, B.W., 2015. Economic Growth: Education vs. Research. J. Glob. Econ. 03.

Klarl, T., 2016. The nexus between housing and GDP re-visited: A wavelet coherence view on housing and GDP for the U.S. Econ. Bull. 36, 704–720.

Kline, S., Rosenberg, N., 1986. An Overview on Innovation, in: Landau, R., Rosenberg, N. (Eds.), The Positive Sum Strateg. National Academy Press. Washington, D.C., USA, pp. 275–305.

Kogut, B., Zander, U., 1992. Knowledge of the firm, combinative capabilities, and the replication of technology. Organ. Sci. 3, 383–397.

Kondratiev, N.K., 1926. Long cycles of economic conjuncture. London Pick. Chato.
Lee, J.W., Lee, H., 2016. Human capital in the long run. J. Dev. Econ. 122, 147–169. https://doi.org/10.1016/j.jdeveco.2016.05.006

Li, Y., Rigby, D., 2022. Relatedness, Complexity, and Economic Growth in Chinese Cities. Int. Reg. Sci. Rev. 0, 1–35. https://doi.org/10.1177/01600176221082308

Lucas, R.E.J., 1988. On the mechanics of economic development. J. Monet. Econ. 22, 3–42.

Mastrogiorgio, M., Gilsing, V., 2016. Innovation through exaptation and its determinants: The role of technological complexity, analogy making & patent scope. Res. Policy 45, 1419–1435. https://doi.org/10.1016/j.respol.2016.04.003

Mewes, L., Broekel, T., 2022. Technological complexity and economic growth of regions. Res. Policy 51, 104156. https://doi.org/10.1016/j.respol.2020.104156

Mowery, D.C., 2010. Technological change and the evolution of the U . S . National Innovation System. 1880 - 1990, in: Innovation: Perspectives for the 21st Century. BBVA, Spain, pp. 129–141.

Nast, C., Broekel, T., Enter, D. (2024). Fueling the Fire? How Government Support Drives Technological Progress and Complexity. Research Policy, 53, https://doi.org/10.1016/j.respol.2024.105005

Nelson, R R, Winter, S., 1982. An evolutionary theory of economic behavior and capabilities, Harvard University Press, Cambridge, MA. Harvard University Press., Cambridge.

Nelson, Richard R, Winter, S.G., 1982. The Schumpeterian tradeoff revisited. Am. Econ. Rev. 72, 114–132.

Nelson, R.R., Wright, G., 1992. The rise and fall of American technological leadership: The postwar era in historical perspective. J. Econ. Lit. 30, 1931–1965.

Niebel, T., 2018. ICT and economic growth – Comparing developing, emerging and developed countries. World Dev. 104, 197–211. https://doi.org/10.1016/j.worlddev.2017.11.024

Oliner, S.D., Sichel, D.E., 2000. The Resurgence of Growth in the Late 1990s : Is Information Technology the. J. Econ. Perspect. 14, 3–22.

Pérez-Balsalobre, S., Llano-Verduras, C., Díaz-Lanchas, J., 2019. Measuring subnational economic complexity : An application with Spanish data. JRC Tech. Rep.

Phene, A., Fladmoe-lindquist, K., Marsh, L., 2006. Breakthrough Innovations in the U.S. Biotechnology Industry: The Effects of Technological Space and Geographic Origin. Strateg. Manag. J. 27, 369–388. https://doi.org/10.1002/smj.522

Pilat, D., 2005. The ICT Productivity Paradox. OECD Econ. Stud. 2004, 37–65. https://doi.org/10.1787/eco_studies-v2004-art3-en

Pinheiro, F.L., Balland, P.-A., Boschma, R., Hartmann, D., 2022. The dark side of the geography of innovation: relatedness, complexity and regional inequality in Europe. Reg. Stud. 1–16. https://doi.org/10.1080/00343404.2022.2106362

Pintar, N., Essletzbichler, J., 2022. Complexity and smart specialization: Comparing and evaluating knowledge complexity measures for European city-regions. Pap. Econ. Geogr. Innov. Stud. 20. https://doi.org/10.13140/RG.2.2.23559.06565

Pintea, M., Thompson, P., 2007. Technological complexity and economic growth. Rev. Econ. Dyn. 10, 276–293. https://doi.org/10.1016/j.red.2006.12.001

Polanyi, M., 1966. Implizites Wissen. Doubledy & Company, Garden City, New York.
Powell, W.W., Giannella, E., 2010. Collective Invention and Inventor Networks, in: Handbook of the Economics of Innovation. Springer, pp. 575–605. https://doi.org/10.1016/S0169-7218(10)01013-0

Pugliese, E., Chiarotti, G.L., Zaccaria, A., Pietronero, L., 2017. Complex economies have a lateral escape from the poverty trap. PLoS One 12, 1–18. https://doi.org/10.1371/journal.pone.0168540

Rescher, N., Michalos, A.C., 1979. Scientific Progress: A Philosophical Essay on the Economics of Research in Natural Science. Phys. Today 32, 55–56. https://doi.org/10.1063/1.2995674

Rigby, D.L., Roesler, C., Kogler, D., Boschma, R., Balland, P.A., 2022. Do EU regions benefit from Smart Specialisation principles? Reg. Stud. 56, 2058–2073. https://doi.org/10.1080/00343404.2022.2032628

Rivkin, J.W., 2000. Imitation of Complex Strategies. Manage. Sci. 46, 824–844.

Romer, P.M., 1990. Endogenous Technological Change. J. Polit. Econ. 98, S71. https://doi.org/10.1086/261725

Saviotti, P.P., 2011. Knowledge, complexity and networks. Handb. Econ. Complex. Technol. Chang.

Saxenian, A., 1994. Regional Advantage - cluture and Competition in Silicon Valley and Route 128. Harvard University Press, Cambridge.

Schmoch, U., Laville, F., Patel, P., Frietsch, R., 2003. Linking technology areas to industrial sectors. Final Rep. to Eur. Comm. DG Res. Karlsruhe, Paris, Bright. 1–71.

Schmookler, J., 1966. Invention and Economic Growth. Harvard University Press, Cambridge.

Šerifi, V., Tarić, M., Jevtić, D., Ristovski, A., Isović, M.Š., 2018. Historical Development of Composite Materials. Ann. ORADEA Univ. Fascicle Manag. Technol. Eng. Volume XXV. https://doi.org/10.15660/auofmte.2018-3.3392

Simon, H.A., 1962. The architecture of complexity. Proc. Am. philosphical Soc. 106, 467–482. https://doi.org/10.1080/14759550302804

Singh, K., 1997. The impact of technological complexity and interfirm cooperation on business survival. Acad. Manag. J. 40, 339–367.

Šmihula, D., Von, C.F., 2010. Waves of technological innovations and the end of the information revolution. J. Econ. Int. Financ. 2, 58–67.

Solow, R.M., 1987. You'd Better Watch out. New York Times, B. Rev. No. 36 114–124. https://doi.org/10.1002/9781444325386.ch10

Solow, R.M., 1956. A Contribution to the Theory of Economic Growth. Q. J. Econ. 70, 65–94.

Sorenson, O., Rivkin, J.W., Fleming, L., 2006. Complexity, networks and knowledge flow. Res. Policy 35, 994–1017.

Stephan, P.E., 2010. The Economics of Science, in: Handbook of the Economics of Innovation. Springer, pp. 217–273. https://doi.org/10.1016/S0169-7218(10)01005-1

Strumsky, D., Lobo, J., Tainter, J.A., 2010. Complexity and the productivity of innovation. Syst. Res. Behav. Sci. 27, 496–509. https://doi.org/10.1002/sres.1057

Sweet, C., Eterovic, D., 2019. Do patent rights matter? 40 years of innovation, complexity and productivity. World Dev. 115, 78–93. https://doi.org/10.1016/j.worlddev.2018.10.009

Teece, D.J., 1988. Technological change and the nature of the firm, in: Dosi, G., Freeman, C., Nelson, R., Silverberg, G., Soete, L. (Eds.), Technical Change and Economic Theory. Pinter Publishers, London.

Teece, D.J., 1986. Profiting from technological innovation: implications for integration, collaboration, licensing and public policy. Res. Policy 15, 285–305.

Teece, D.J., Pisano, G., Shuen, A., Jose, S., 1997. Dynamic Capabilities and Strategic Management. Strateg. Manag. J. 18, 509–533.

Teixeira, A.A.C., Queirós, A.S.S., 2016. Economic growth, human capital and structural change: A dynamic panel data analysis. Res. Policy 45, 1636–1648. https://doi.org/10.1016/j.respol.2016.04.006

Torrence, C., Webster, P.J., 1999. Interdecadal Changes in the ENSO–Monsoon System. J. Clim. 12, 2679–2690. https://doi.org/10.1175/1520-0442(1999)012<2679:ICITEM>2.0.CO;2

Uzzi, B., Mukherjee, S., Stringer, M., Jones, B., 2013. Atypical combinations and scientific impact. Science (80-.). 342, 468–472. https://doi.org/10.1126/science.1240474

Vaesen, K., Houkes, W., 2017. Complexity and technological evolution: What everybody knows? Biol. Philos. 32, 1245–1268. https://doi.org/10.1007/s10539-017-9603-1

Valverde, S., Sole, R. V., 2015. Punctuated equilibrium in the large-scale evolution of programming languages. J. R. Soc. Interface 12. https://doi.org/10.1098/rsif.2015.0249

van Ark, B., Inklaar, R., McGuckin, R.H., 2003. ICT and Productivity in Europe and the United States Where Do the Differences Come From? CESifo Econ. Stud. 49, 295–318. https://doi.org/10.1093/cesifo/49.3.295

van der Wouden, F., 2020. A history of collaboration in US invention: changing patterns of co-invention, complexity and geography. Ind. Corp. Chang. 29, 599–619. https://doi.org/10.1093/icc/dtz058

Verona, F., 2020. Investment, Tobin's Q, and Cash Flow Across Time and Frequencies. Oxf. Bull. Econ. Stat. 82, 331–346. https://doi.org/10.1111/obes.12321

Waters, K., Shutters, S.T., 2022. Impacts of Skill Centrality on Regional Economic Productivity and Occupational Income. Complexity 2022. https://doi.org/10.1155/2022/5820050

Youn, H., Bettencourt, L.M.A., Lobo, J., Strumsky, D., Samaniego, H., West, G.B., 2016. Scaling and universality in urban economic diversification. J. R. Soc. Interface 13, 20150937. https://doi.org/10.1098/rsif.2015.0937

Zander, U., Kogut, B., 1995. Knowledge and the Speed of the Transfer and Imitation of Organizational Capabilities: An Empirical Test. Organ. Sci. 6, 76–92.

Zou, Y., Huang, M., Xiang, W., Lu, L., Lu, Y., Gao, J., Cheng, Y., 2022. The impact of high-tech industry development on energy efficiency and its influencing mechanisms. Front. Environ. Sci. 10, 1–19. https://doi.org/10.3389/fenvs.2022.962627

Appendix

A.1 Continuous Wavelet Analysis: Technical details

We provides a short introduction to the main elements of the CWT framework that is used in the paper's empirical analysis. The starting point of the CWT framework is to search for a function that fulfills some properties for being used as a mother (or analyzing) wavelet, $\varphi(t)$. The choice is a delicate task and context-specific. Crucially, the function must be well-localized in time and have zero mean. Under these conditions, this function can be seen as a small "wave" (hence, the term wavelet) rapidly losing strength with decreasing distance from the time-frequency center[11].

[11] The interested reader is refered to Aguiar-Conraria and Soares (2014) sections 2.3 and 2.9.1 for technical details about admissible wavelets.

We use the most popular wavelet, the so-called Morlet (or Gabor) wavelet, whose characteristics are optimal to deal with such patterns. It can be written as $\varphi(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2}$, with ω_0 which is the wavelet's central angular frequency that is commonly set to *6* (Aguiar-Conraria and Soares, 2014; Flor and Klarl, 2017). This specification greatly simplifies the interpretation of our empirical results.[12] The CWT maps the original time series, i.e., economic growth termed as $y(t) \in \mathcal{L}^2(\mathbb{R})$, into a function of time and frequency[13] ($W_y(\tau, s)$) using the two parameters τ and s:

$$W_y(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{+\infty} y(t) \widehat{\varphi} \left(\frac{t-\tau}{s}\right) dt, \qquad (1)$$

whereby s is a scaling factor that governs the length of the wavelet, and τ is a translation parameter controlling the location of the wavelet. If $|s| < 1$, the length of the wavelet is compressed to measure short-run cycles (high frequency), while in case of $|s| > 1$, the length of the wavelet is stretched to measure long-run cycles.

As $\widehat{\varphi}(\cdot)$ denotes the complex conjugation of $\varphi(t)$, the CWT in (1) is also complex-valued and can be separated into its imaginary part, $\mathfrak{I}(W_y(\tau, s))$ and real part $\mathfrak{I}(W_y(\tau, s))$.
The advantage of CWT is that we can directly obtain the phase and phase difference of the wavelet transform (the set of wavelets and their frequencies describing the time series) of each time series (such as complexity). In formal terms, the phase of an individual time series *y(t)* is computed as p_y= arctan $\left(\frac{\mathfrak{I}(W_y(\tau,s))}{\mathfrak{R}(W_y(\tau,s))}\right)$. However, we are particularly interested in computing the phase difference as it describes the relationship between the two time-series, say *x(t)* and *y(t)*. For this purpose, we first have to compute the so-called cross-wavelet transformation of the two time series $\{x(t), y(t)\} \in \mathcal{L}^2(\mathbb{R})$ (see Hudgins et al., 1993) as $W_{xy} \equiv W_{xy}(\tau, s) = W_x(\tau, s)\widehat{W_y}(\tau, s)$, where $W_x(\tau, s)$ and $W_y(\tau, s)$ are the CWT of a time series *x(t)* and *y(t)*, respectively. In a second step, the *phase difference* for two time series, say $\{x(t)\ and\ y(t)\} \in \mathcal{L}^2(\mathbb{R})$ can be computed as follows:

$$\phi_{x,y} = \arctan\left(\frac{\mathfrak{I}\left(W_{xy}(\tau,s)\right)}{\mathfrak{R}\left(W_{xy}(\tau,s)\right)}\right) \in [-\pi, \pi], \qquad (2)$$

where, for a given complex number z, $\mathfrak{I}(z)$ and $\mathfrak{R}(z)$ denote its imaginary and real part, respectively. (2) is also equal to the phase difference in the angular form $\phi_{x,y} = \phi_x - \phi_y$. Intuitively, if the phase difference is zero, both time series move together because they have the same phase angle. If the phase difference is different from zero, two variables *x(t)* and *y(t)* move either in or out-of-phase, while one variable either leads or lags the other. Hence, it provides information on the direction of the effects and potential lead-lag patterns.
Abbreviating *x(t)=x* and *y(t)=y*, for $\phi_{x,y} \in \left[0, \frac{\pi}{2}\right]$, the two time series move in-phase, where x is leading y; for $\phi_{x,y} \in \left[-\frac{\pi}{2}, 0\right]$, where y is leading x. For $\phi_{x,y} \in \left[\frac{\pi}{2}, \pi\right]$, the two time series move out-of- phase, where y leads x, while for $\phi_{x,y} \in \left[-\pi, -\frac{\pi}{2}\right]$, x is leading y. In other words, for the complexity and growth time series, performing this computation yields information about possible delays of the oscillations of their time series.

[12] To be more precise, for a given frequency f and scale s, we have $f = \frac{6}{2\pi s} \approx \frac{1}{s}$ (see Cazelles et al., 2008; Torrence and Webster, 1999)
[13] The squared norm of $y(t)$ is referred to as the energy, while the space $\mathcal{L}^2(\mathbb{R})$ is commonly known as the space of finite energy functions.

Our econometric approach features more than two explanatory variables, therefore requiring a multivariate extension. In other words, we introduce wavelet tools that can be used to explore co-movements between complexity and growth by controlling for other influencing factors. We rely on the *partial wavelet coherency (PWC)*, which, in essence, give insights into the statistical significance of the identified effects and lead-lag patterns. Given a vector of time series $X(t) = [x_1(t)', x_2(t)', x_3(t)', \dots, x_q(t)']$,[14] the PWC between, say $x_i(t)$ and $x_j(t)$ with $1 \leq i \leq p, 1 \leq j \leq p$, after controlling the influence of all other variables in $X(t)$, reads as

$$\breve{R}_{1\ j.\bar{q}_j} \equiv \frac{|\mathcal{M}_{j1}^d|}{\sqrt{\mathcal{M}_{11}^d}\sqrt{\mathcal{M}_{jj}^d}},$$

with $\bar{q}_j = \{2, \dots, p\}\{j\}$ for $2 \leq j \leq p$. $\mathcal{M}$ is a $(q \times q)$ Hermitian matrix of all smoothed cross-wavelet spectra, i.e. $\mathcal{M}_{ij}^d = (S_{ij})_{i,j=1}^{q}$, where S_{ij} is the smoothed version of the cross-wavelet spectrum W_{ij} of two series $x_i(t)$ and $x_j(t)$. $\mathcal{M}_{ij}^d$ is the co-factor of the element (i, j) of $\mathcal{M}$, that is $\mathcal{M}_{ij}^d = (-1)^{(i+j)} \det(\mathcal{M}_i^j)$.

Moreover, we refer to Mandler and Scharnagl and compute the so-called *partial wavelet gain* (PWG) of a variable $x_1(t)$ and $x_2(t)$ after controlling the influence of all other variables in $X(t)$ as follows:

$$\mathcal{G}_{1\ j.\bar{q}_j} = \breve{R}_{1\ j.\bar{q}_j} \frac{\sqrt{\mathcal{M}_{jj}^d}}{\sqrt{\mathcal{M}_{11}^d}},$$

The *partial wavelet gain* $\mathcal{G}_{12\cdot3\ldots q}$ is directly associated with textbook econometrics: It can be interpreted the coefficient in the multiple linear regression of $x_1(t)$ on the set of $q-1$ explanatory variables $x_2(t), x_2(t), \dots, x_q(t)$ at each time and frequency. Consequently, it describes the strength of one time series impacting the other.

Example: Three variable case

Let's assume that we want to explore the Granger-causal relationship between complexity and growth by controlling for the number of patents. This corresponds to a three variables case ($q = 3$). For these three time series x_1, x_2 and x_3, the matrix of complex coherencies is given by

$$\mathcal{M} = \begin{bmatrix} 1 & m_{12} & m_{13} \\ m_{21} & 1 & m_{23} \\ m_{31} & m_{32} & 1 \end{bmatrix}.$$

To compute the partial wavelet coherence, we have to compute the following co-factors of $\mathcal{M}$: $\mathcal{M}_{11}^d = \begin{vmatrix} 1 & m_{23} \\ m_{32} & 1 \end{vmatrix}$=1-$m_{32}m_{23} = 1 - R_{23}^2$, $\mathcal{M}_{22}^d = 1 - R_{13}^2$ and $\mathcal{M}_{21}^d = (-1)^{1+2} \begin{vmatrix} m_{12} & m_{13} \\ m_{32} & 1 \end{vmatrix} = -(m_{12} - m_{13}m_{32})$. Thus, for the three variable case, the partial wavelet coherence reads as:

[14] Note the change in the notation: $y(t) \equiv x_1(t)$.

$$\breve{R}_{12.3} \equiv \frac{|m_{12} - m_{13}m_{32}|}{\sqrt{1-R_{23}^2}\sqrt{1-R_{13}^2}},$$

while the partial wavelet gain matrix between x_1 and x_2 (after controlling for x_3) for different times and frequencies is given by:

$$\mathcal{G}_{12.3} \equiv \breve{R}_{12.3} \frac{\sqrt{1-R_{13}^2}}{\sqrt{1-R_{23}^2}}.$$

The example shows that $\mathcal{G}_{12 \cdot 3}$ corresponds to a regression coefficient matrix by regressing x_1 on x_2 and x_3 at each time and frequency in the multiple linear regression model framework.

Moreover, we rely on Torrence and Compo (1998) to test for the significance of the wavelet power spectrum. Regarding coherency and partial coherency, we use Monte-Carlo simulations for the significance tests. In our case, we fit an ARMA (1,1) model to each series and draw new samples by drawing from a Gaussian distribution with a corresponding variance that is equal to that of the estimated standard errors. We repeat these exercises several times (5,000 replications) and extract critical values. We have also experimented with higher orders of the ARMA ($\hat{p}$, $\hat{q}$)model and have also increased the number of replications to a maximum value of 10,000. These changes do not significantly affect our results.

A.2 Robustness checks

Figure 13 shows the results of the wavelet analysis when calculating the complexity value as the median of the 1%, 10%, and 50% (Median) percentiles complexity frontiers. The results for the 1% and 10% percentiles, by and large, confirm those obtained using the 5% percentile. However, there are some important differences; the first one is that the area of significance in 1890-1907 is shaped a bit differently when using the 1% frontier definition, whereas the 5% and 10% specification return rather similar results. Noticeably, a second "heat spot" emerges when looking at the 1% specification, which also starts around 1890 but lasts until 1920, with a temporal variance of 24-27 years. However, this pattern only appears in this specification, and the corresponding wavelet gain is rather small (around 0.3%). In any case, it suggests that in the period in which GDP growth drove complexity (the 1890s to the 1910s), this was not limited to the most complex technologies but encompassed the wider set of highly complex technologies. The results for the median complexity (Complexity Top 50% in the third row in Figure 13) add to this with a strongly significant relationship between economic growth and median complexity between 1880 and 1910. The relationship is restricted to a 15-17 years-temporal variance in this case.

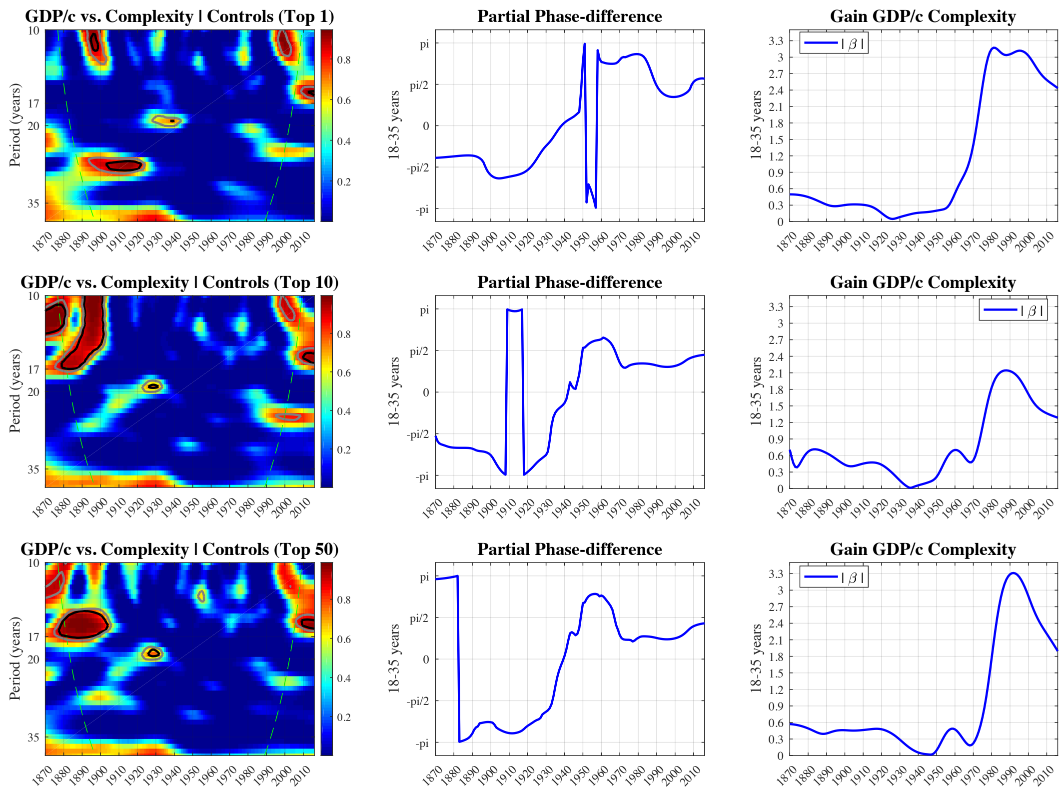

Figure 13: Alternative complexity frontier specifications (1%, 10%, and 50% most complex) and GDP/c growth

The second most noticeable difference concerns the period 1990 to 2011, during which we established a significant relationship between complexity and economic growth using the 5% percentile frontier. This relationship is similarly visible in the 1% specification (although not as clearly). It does not become significant using the 10% and 50% percentiles of complexity specifications. Consequently, this relationship is linked to the most complex technologies, i.e., the frontier, driving economic growth.

In sum, small variations in the percentiles considered in constructing the complexity frontier do not challenge our main results. However, moving away from the 5% threshold frontier (small percentiles) towards the median particularly weakens the statistical relationship between complexity and economic growth after 1990. This supports the idea that the complexity frontier is relevant for growth, not the medium complexity of used technologies at a certain time.

A.3 Original complexity frontier

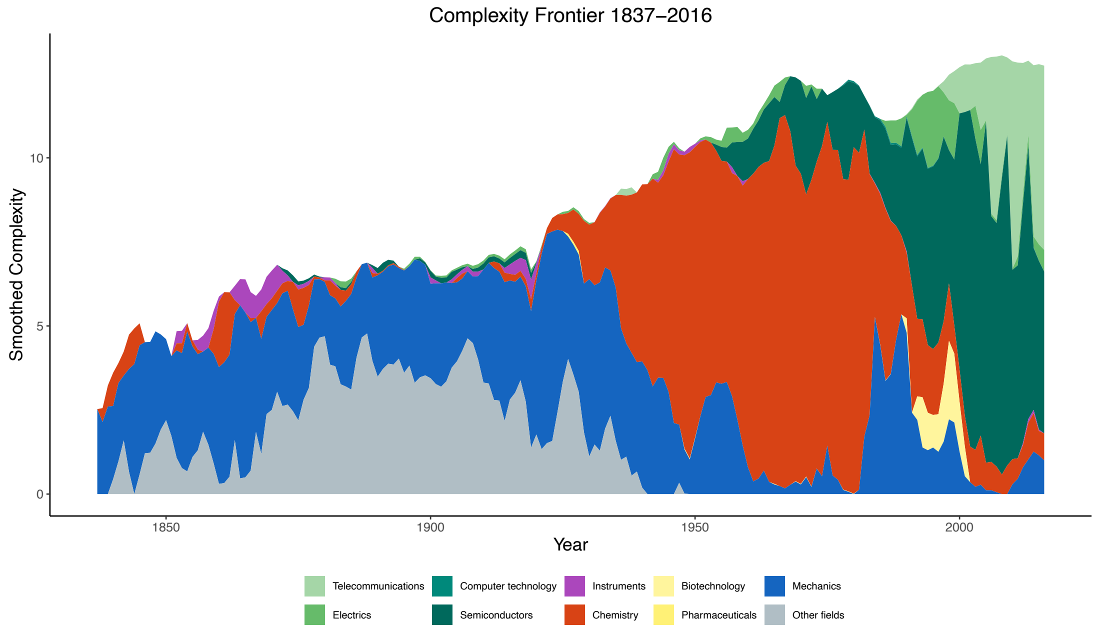

Figure 13: Original complexity frontier by sector (no smoothing)

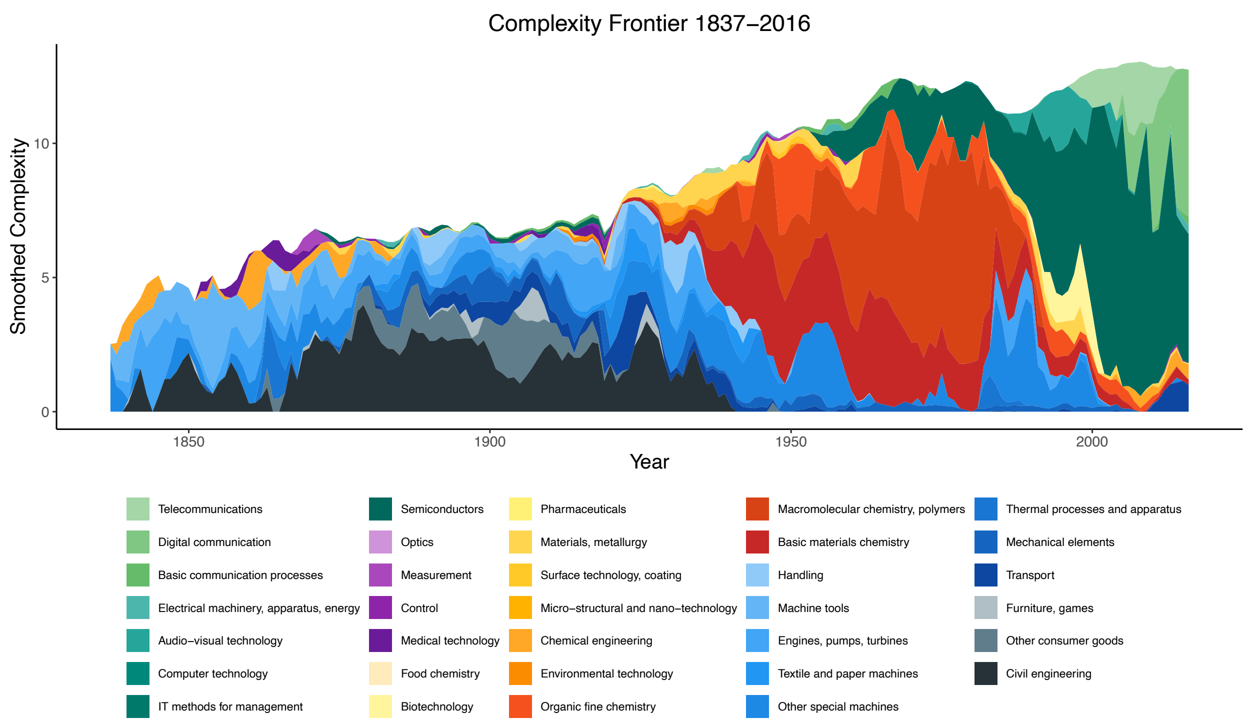

Figure 14: Original complexity frontier by technological field (no smoothing)