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The geography and evolution of complex knowledge

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Abstract: There is consensus among scholars and policy makers that knowledge is one of the key drivers of long-run economic growth. It is also clear from the literature that not all knowledge has the same value. However, too often in economic geography and cognate fields we have been obsessed with counting knowledge inputs and outputs rather than assessing the quality of knowledge produced. In this paper we measure the complexity of knowledge across patent classes and we map the distribution and the evolution of knowledge complexity across U.S. cities from 1975 to 2004. We build on the 2-mode structural network analysis proposed by Hidalgo and Hausmann (2009) to develop a knowledge complexity index (KCI) for Metropolitan Statistical Areas (MSAs). The KCI is based on more than 2 million patent records from the USPTO, and combines information on the technological structure of 366 MSAs with the 2-mode network that connects cities to the 438 primary (USPTO) technology classes in which they have Relative Technological Advantage (RTA). The complexity of the knowledge structure of cities is based on the range and ubiquity of the technologies they develop. The KCI indicates whether the knowledge generated in a given city can be produced in many other places, or if it is so sophisticated that it can be produced only in a few select locations. We find that knowledge complexity is unevenly distributed across the U.S. and that cities with the most complex technological structures are not necessarily those that produce most patents.

Key words: knowledge complexity, cities, patents, network analysis, economic geography, United States

JEL codes: O33, R11, L65, D83

1. Introduction

"What is important in knowledge is not quantity, but quality. It is important to know what knowledge is significant, what is less so, and what is trivial" (Leo Tolstoy; a calendar of wisdom).

It has become commonplace to regard the production of knowledge and the diffusion of that knowledge across space as key to understanding the uneven geography of growth and development (Schumpeter 1942; Solow 1956; Nelson and Winter 1982; Romer 1990; Corrado and Hulten 2010; OECD 2013). While knowledge has always been a critical input to production, the centrality of its role to capitalist competition has grown recently as transport costs for a wide variety of commodities have been lowered and as global commodity markets have been increasingly integrated (Dunning 2002; Dicken 2007). This does not mean that the usual foundations of profitability have been flattened, that there are no longer lower cost sites of production for particular goods, or richer markets, but rather that a growing number of firms from around the world have increased access to such sites for more and more segments of their value. Within this environment, knowledge that is spatially "sticky", difficult to create or to move outside the region of its production, has taken on added value (Lundvall and Johnson 1994; Markusen 1996; Gertler 2003). For many firms and regions of the industrialized world, competitive advantage hinges on the production of high-value, non-ubiquitous, complex and tacit knowledge (Maskell and Malmberg 1999; Asheim and Gertler 2005).

Why are some cities and regions more innovative or creative than others? Because of the importance of knowledge in contemporary capitalism, and because of the role of cities in its production, researchers in economics, geography, science and innovation studies as well as local policy makers have focused attention on this question. It is crucial to identify differences in the nature and the pace of innovation between cities to design efficient knowledge-based policy. However, too often in the literature we have been obsessed with counting knowledge outputs rather than assessing the quality of knowledge produced. In this paper, we separate the quantity and quality of knowledge production by mapping the distribution and evolution of (technological) knowledge complexity in U.S. cities from 1975 to 2004. We build on the 2-mode structural network analysis proposed by Hidalgo and Hausmann (2009) to develop a knowledge complexity index (KCI) for Metropolitan Statistical Areas (MSAs). The KCI is based on more than 2 million patent records from the United States Patent and Trademark Office (USPTO), and combines information on the technological structure of 366 MSAs with the 2-mode network that connects cities to the 438 technologies in which they have Relative Technological Advantage (RTA). Following this network approach, we characterize the complexity of the knowledge structure of cities based on the range and ubiquity of the technologies they develop. The KCI indicates whether the knowledge embodied in a given city can easily be (re)produced in many other MSAs, or if it is so sophisticated that it can only be produced by a few key cities. We find that knowledge complexity is unevenly distributed in the U.S. and that cities with the most complex technological structure are not necessarily the ones with the highest rates of patents per capita. Our results suggest that looking at knowledge quality on top of knowledge quantity provides insights on the distribution of knowledge production that cannot be captured by simply counting aggregate knowledge outputs such as patents.

The rest of the paper is organized as follows. In section 2 a brief review of relevant literature is provided. Section 3 describes construction of the *city-tech knowledge network* from patent data, the analytical backbone of our methodological framework. The structural analysis of this network and the underlying principles of the knowledge complexity index are discussed in Section 4. Section 5 presents empirical evidence on the geography and evolution of knowledge complexity in U.S. cities. Section 6 offers some preliminary conclusions and directions for future research.

2. Literature review

Economic geographers have long recognized geographical patterns of specialization in the distribution of industries (Scott 1996; Ellison and Glaeser 1999), in techniques of production (Rigby and Essletzbichler 1997; 2006), in organizational and institutional formations (Saxenian 1994; Storper 1997), and in research and development (Audretsch and Feldman 1996). That subsets of knowledge, or technological know-how, emerge in different places is strong evidence of the existence of localized communities of practice (Lawson and Lorenz, 1999) that reflect place-specific sets of technological competences, capabilities and institutional relations (Storper 1993; Gertler 1995; Boschma and Frenken 2007).These capabilities are often built up over long periods of time and they shape the environment within which subsequent choices are made (Essletzbichler and Rigby 2007). Grabher (1993) argues that the path dependent nature of economic evolution locks some regions into technological regimes that yield diminishing returns, while Saxenian (1994) provides compelling evidence of regional variations in the capacity to maintain innovation. Long-run creativity within regions is linked to institutional practices that foster open knowledge architectures, absorptive capacity and connections to pools of knowledge generated elsewhere (Cohen and Levintahl 1990; Bathelt et al., 2004; Asheim and Coenen 2005).

The persistence of regional differences in knowledge-bases suggests not only that invention is cumulative in nature, resulting from the recombination of existing ideas and from processes of search that tend to be localized, but also that knowledge subsets developed in one location are often difficult to replicate elsewhere. David (1975) and Nelson and Winter (1982) argue that the cumulative nature of much technological change is limited by the sunk costs of accumulating experience. These claims are reinforced by models of search in which costs of exploration rise steeply outside the boundaries of familiar knowledge terrain (Atkinson and Stiglitz 1969; Binswanger 1974; Stuart and Podolny 1996; Antonelli 2005). In turn, these ideas have helped popularize the image of knowledge development as a process of recombination (Evenson and Kislev 1976; Weitzman 1998). Olsson and Frey (2002), and many others, build on the fitness landscapes of Kauffman (1993) to argue that successful recombination is related to the number of ideas in knowledge space, the distance and the extent of the interaction between them.

The difficulties of moving certain kinds of knowledge are discussed by Kogut and Zander (1992), by Lundvall (1988) and Gertler (1995). Kogut and Zander (1992) offer a knowledge-based view of the firm as an organizational unit adapted to replicating knowledge while limiting its imitation by competitors. Defining knowledge as technological know-how (see also von Hippel 1988), they envision the firm as a coherent set of organizing principles, similar to the routines of Cyert and March (1963) and Nelson and Winter (1982), that link and combine complex and tacit knowledge held by skilled workers in collective sets of procedures, that often themselves embody a tacit dimension. When these routines are shared across economic agents in agglomerations that are united by traded and untraded inter-dependencies (Marshall 1920; Storper 1995), so our conception of the knowledge-based region emerges (Lundvall and Johnson 1994; Tallman et al. 2004; Asheim and Gertler 2005). In both these visions, knowledge-based firms and knowledge-based regions are more than simply the sum of their (knowledge) parts. In an all too often used aphorism, adulterated from Polanyi (1966), regions, like firms, know more than they can tell.

Though considerable theoretical effort has been directed towards uncovering what it takes to be a learning region or a knowledge economy, much less attention has been given to the character of knowledge produced within regions. One of the primary reasons we know so little about the spatial composition of knowledge is that we lack precise measures of knowledge and technology (Pavitt, 1982). Recent work has attempted to capture differences in the nature of knowledge cores over space. Inspired by measures of the technological distance between firms (Jaffe 1986) and measures of technological coherence (Teece et al. 1994), Graff (2007), Kogler et al. (2013) and Rigby (2013) use patent data to measure distances between classes of technologies and provide visualizations of national and local knowledge spaces and their evolution over time. Balland et al. (2014) and Rigby (2013) explore how the structure of these spaces guide localized trajectories of knowledge development through patterns of technological abandonment and diversification following the product-space arguments of Hidalgo et al. (2007) and work on relatedness (Neffke 2009). Extensions of these same ideas underpin models of knowledge flow between regions (Jaffe et al. 1993; Fischer et al. 2006) that is linked to geographical, social and cognitive proximity (Jaffe et al. 1993; Breschi et al. 2003; Feldman et al 2013). Boschma (2005) reviews these arguments.

Patent data are also used to measure the wealth of regions from a knowledge perspective. Acknowledging the standard criticisms of patent data (Griliches 1990), regional knowledge stocks can be generated through perpetual inventory techniques, counting patents by their geography and using the length of patent protection as an indicator of the "service life" of knowledge. However, such simple accounting procedures pay little attention to the heterogeneity of the knowledge embodied within individual patents and thus to patent values. That inventions differ in their capacity to punctuate the incremental nature of much technological advance is broadly understood (Sahal 1981, Dosi 1982; Abernathy and Clark 1985; Christensen 1997). Fortunately, there have been numerous attempts to assess the quality and the value of individual patents. Trajtenberg (1990) measures patent values through forward citations and ties those measures to social valuations of important innovations within the computer tomography sector. Hall et al. (2005) combine patent records with COMPUSTAT firm data and show the correlation between citation-weighted patent counts and the market value of firms. Harhoff et al. (1999) survey German patent holders and find a strong correlation between the citation value of patents and estimates of the price at which they would be willing to sell patents shortly after filing. Lerner (1994) links patent scope, the breadth of knowledge claims, to the value of assignee firms. Harhoff et al. (2003) and Lanjouw and Schankerman (2004) use citations, family-size, renewals and litigation in composite measures of patent value. Ejermo (2009) employs similar methods to weight patent counts across Swedish regions. In related research, Schoenmakers and Duysters (2010) trace the technological origins of blockbuster patents to the number of knowledge domains they combine. Kelley et al. (2013) use a similar definition in their examination of breakthrough patents in the drug and semiconductor sectors. Verspagen (2007) also uses patents and citation data related to fuel cells to uncover critical branching points in knowledge development that steer subsequent trajectories of technological development. Castaldi et al. (2013) explore the geography of breakthrough patents in the United States.

While patent valuations provide one indicator of the value of knowledge held by firms and located in different regions, another critical dimension of the competitive advantage conveyed by knowledge is its inimitability. Nickerson et al. (2007) argue that both value creation and capture sit at the core of strategic management theory and the knowledge-based view of the firm. This raises the question of what makes knowledge more or less difficult to replicate. For Simon (1962), the complexity of different knowledge architectures influence their potential exclusivity and value. He defines complex systems as comprising large numbers of components that interact in non-simple ways and that are often non-decomposable. Kauffman's (1993) fitness landscapes are defined across similar dimensions. For Kauffman, the higher the interaction among a set of components, the more rugged the search landscape, the higher the cost of search and the more valuable the optimal solution. We might think about knowledge-based firms and regions in the same fashion: they comprise many components that interact in non-trivial ways to produce high-value solutions to complex problems. And, it is this complexity that aids value capture by rendering tacit much of what they do. Indeed, it is the tacit dimension of complex knowledge production that makes it so difficult to move between firms and between regions (Gertler 2003; Nickerson and Zenger 2004) and that makes it valuable (Maskell and Malmberg 1999). Recognition that complex and tacit knowledge is relatively immobile has spurred a number of papers on knowledge sourcing. Almeida (1996) uses field interviews and patent citation data to explore local and non-local knowledge sourcing in the U.S. semi-conductor industry. He reports that innovative regions act like magnets to foreign multi-nationals, especially when technological knowledge is perceived to be sticky. Chung and Alcacer (2002) confirm that in research-intensive sectors, foreign firms from both technologically leading and lagging nations are attracted by R&D spending in U.S. states. Cantwell and Piscitello (2002) reveal that foreign multi-nationals in Europe are more likely to locate their foreign research plants within regions that have attractive knowledge-bases. Tempering these claims, Singh (2008) shows that MNCs with geographically distributed R&D activities have lower quality innovations, and suggests that this likely results from the difficulties of integrating knowledge from different sources. Todtling et al. (2011), building on earlier work by Trippl et al. (2009), explore local knowledge sourcing within the ICT sector across a large and a small metropolitan region of Austria. They report that the structure of local knowledge networks shapes the patterns of knowledge access and the types of knowledge acquired.

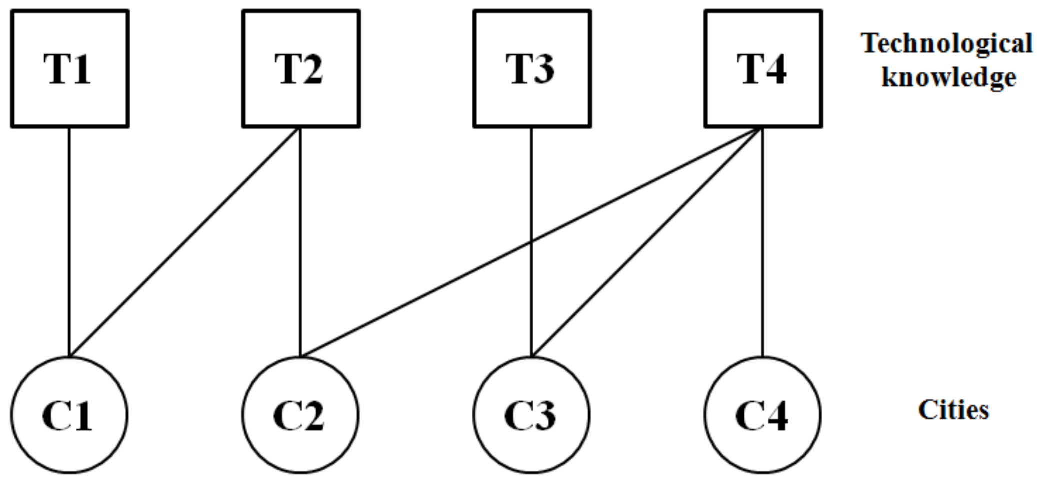
Which regions hold the most valuable knowledge, especially knowledge that is complex and tacit and thus difficult to access? So far it has proven difficult to answer this question, at least in part because we have no readily available measures of the complexity or the tacit nature of knowledge located in particular places. Developing the work of Kauffman (1993) within the management literature, and extending the arguments of Levinthal (1997) and Rivkin (2000), Fleming and Sorenson (2001) offer a model of search-based, recombinant innovation that rests on the complexity of knowledge. Using patent data, they provide a measure of complexity using estimates of the difficulty of combining knowledge subsets represented by different technology sub-classes in USPTO data. In a subsequent paper, Sorenson (2005) links measures of the complexity of patent classes to industries and investigates the relationship between industry agglomeration and the complexity of industrial knowledge. He shows that when industrial knowledge complexity increases, social networks play a critical role in knowledge transfer and learning and the centripetal forces of such networks maintain agglomeration. When knowledge complexity is lower and social networks less important to technology flows and learning, industries are more likely to disperse. In work on the product-space of countries, Hidalgo and Hausmann (2009) develop a different measure of product and place complexity based on the product-level diversity of national economies and the ubiquity (or range) of countries across which individual products are produced. To date, no one has used either of these techniques to examine the complexity of knowledge located in cities and regions of the U.S., or most anywhere else.

3. The city-tech knowledge network

The analytical backbone of this framework is the *city-tech knowledge network* that connects cities to the technological knowledge they develop. This is a 2-mode network (Borgatti, 2009), the structure of which emerges out of the connections between nodes of different types[1], in this case between cities and technologies (see Figure 1). This type of network is also referred to as a bipartite, bimodal or an affiliation network in the network science literature (Opsahl, 2013). Typical examples of 2-mode networks are individual-event networks (Davis et al., 1941), interlocking directorates (Robins and Alexander, 2004), predator-prey networks (Allesina and Tang, 2012) or firm-projects networks

[1] Connections between nodes of the same mode, i.e. city-city or technology-technology ties are not considered.

(Balland, 2012). Although we focus on a network of cities and technologies, the structural analysis of 2-mode networks formed by other types of spatial units and knowledge domains offer various ways for understanding geographies of innovation. Following Hidalgo and Hausmann (2009), we show that the particular architecture of the *city-tech network* reveals the relative capacity of cities to produce complex technological knowledge.



Note: The connections represent the production of technological knowledge "T" by city "C".

Figure 1. The (2-mode) city-tech network

To construct the *city-tech knowledge network* we use patent documents from the United States Patent and Trademark Office (USPTO) from 1975 to 2004. The connections between cities and technologies are established over time as inventors within cities develop new knowledge (patents) in given technological fields[2]. Patent data provide precise and systematic information on the production of knowledge in different technology fields (the first set of nodes in Figure 1) over space (the second set of nodes in Figure 1) and time. These are crucial inputs for construction of the 2-mode network.

Since we are interested in the timing of new knowledge creation we use the application year of the patent and not the grant year because of the variable time-lag that the examination process entails. It is in the process of examination that each (granted) patent is allocated to one or more

[2] We only focus on the complexity of technological knowledge produced in cities. We do not consider artistic, cultural or other forms of knowledge. We also recognize that not all new technological knowledge is captured by patents.

distinct technology classes that reflect the technological characteristics of the new knowledge created. By the end of 2004, there were 438 primary technology classes of utility patents in use by the USPTO (see Strumsky et al., 2012). In this paper, we allocate individual patents to their primary technological class only. Patent documents also provide information on the place of knowledge production by referencing the home address of inventors. We only consider patents produced by inventors located within the United States, and in the case of co-invention, patents are located by the address of the first-named, primary inventor. We discard patent records if the primary inventor is not located in one of the 366 U.S. metropolitan areas.

More formally, we represent the geography of technological knowledge production as an n by k 2-mode adjacency matrix. The resulting network involves $n=366$ cities (MSAs) and $k=438$ technological domains or classes. In this $n*k$ matrix, the weight of each edge $x_{c,i}$ is the number of patents produced within city c in technological category i $(c = 1,..., n;\ i = 1,\ ...,\ k)$. We divide the years for which we have patent data, 1975-2004, into six periods of five years, and we construct a 2-mode city-knowledge network for each of these periods. Figure 2 shows a visual representation[3] of the *city-tech knowledge network* for the latest period. For clarity, the network visualization presented in Figure 2 does not show the full 2-mode network structure, but rather a summary of its structure using a maximum spanning tree algorithm. The maximum spanning tree T of the $n*k$ *city-tech knowledge network* is the $n*k$ sub-graph with $(n+k-1)$ edges which has a maximum total weight. This is the backbone of the network. Two rules apply: (1) the network should stay fully connected, i.e. no isolates (cities or technologies) should be generated while removing the links, and (2) the sum of the weight of the links of the sub-graph should be the highest possible. Of course, this graph representation only gives a general idea of the *city-tech knowledge network* and the structural analysis presented in the next section is based on the full network.

[3] This graph has been visualized using the Gephi software.

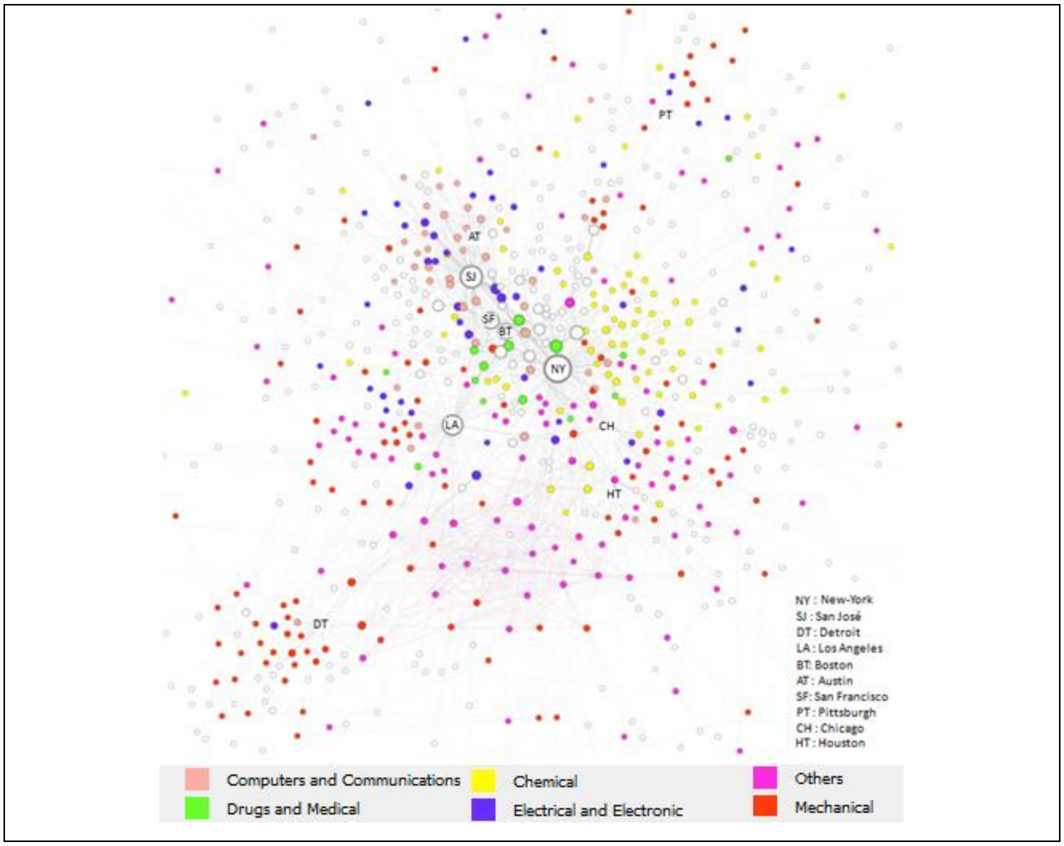


Figure 2. The structure of the city-tech knowledge network

In Figure 2, cities are represented by white nodes, while technologies are represented by colored nodes. Each color corresponds to one of seven aggregate patent categories identified by Hall et al. (1999). The position of cities in the knowledge space reflects the technological classes in which they have RTA as well as the density of their patents across these classes. Figure 2 shows, for instance, the specialization of San-José in computers & communications and electrical & electronics technologies, along with the smaller ICT/electronic hub in Austin. San Francisco is pulled a little away from those technologies toward biotech and pharma more generally. More diversified cities like New York and Boston occupy the center of the knowledge space, a region where links between technology nodes are particularly dense and where there are strong possibilities for technological recombination as existing competences can be readily redeployed. Los Angeles and Chicago are only slightly removed from the knowledge core. Detroit occupies a more peripheral location, embedded in a mechanical cluster within the knowledge space, a cluster that is by now somewhat less well-connected to other technologies. Houston and Pittsburgh also occupy somewhat more peripheral parts of knowledge space.

As is often the case in complex networks research, the visual representation of the 2-mode network is limited by the number of nodes and ties that can be identified. Despite major advances in layout algorithms for large scale networks, visualization can only offer preliminary insights into the structure of the *city-tech knowledge network.* We turn to a more comprehensive, statistical analysis of that structure below.

4. Knowledge complexity index

Simultaneously combining information on (1) which cities produce specific technologies and (2) how common specific technologies are across cities, it is possible to construct an indicator that captures the level of knowledge complexity of a city's technological portfolio for a given period of time. This knowledge complexity index (KCI) is based on the "method of reflections" developed by Hidalgo and Hausmann (2009). In their pioneering work, Hidalgo and Hausmann show that the economic complexity of a country's output is reflected by the particular composition of its export basket, taking into account the relative composition of the export baskets of all other countries. The main idea in their analytical framework is that more complex economies produce more exclusive goods, i.e. non-ubiquitous commodities that are sourced in relatively few countries in total. Countries with complex economic structures experience a privileged source of comparative advantage, a form of spatial-technological-monopoly from which they extract rents. Countries that produce goods that are widely imitated by others, commodities that are ubiquitous, tend to have low scores in terms of economic complexity. Following this approach, we analyze the particular architecture of the *city-tech knowledge network* and we show that a city has a complex technological composition if it produces knowledge that relatively few other cities are able to imitate.

To construct our index of knowledge complexity, we only consider cities that are significant producer of a particular technological knowledge. As a result, it should be noted that the city-tech knowledge network that is used to compute the KCI is based only on technological classes in which a city has a relative technological advantage (RTA) in terms of patenting activity. The network can be represented as a $n*k$ 2-mode adjacency matrix $\boldsymbol{M} = (M_{c,i})$, where $M_{c,i}$ reflects whether or not city c has RTA in the production of technological knowledge i $(c = 1,\ldots, n;\ i = 1, \ldots, k)$. A city c has RTA in technology i at time t if the share of technology i in the city's technological portfolio is higher than the share of technology i in the entire U.S. patent portfolio. More formally, $RTA_{c,i}^{t} = 1$ if:

$$\frac{patents_{c,i}^{t} / \sum_i patents_{c,i}^{t}}{\sum_c patents_{c,i}^{t} / \sum_c \sum_i patents_{c,i}^{t}} > 1$$

Following the method of reflections, the KCI sequentially combines two variables: the diversity of cities and the ubiquity of technological classes. These two variables correspond to the 2-mode degree centrality of both sets of nodes in the city-tech knowledge network. The degree centrality of cities ($k_{c,0}$) is given by the number of technological classes in which each city has RTA (diversity):

$$DIVERSITY = K_{c,0} = \sum_i M_{c,i} \quad (1)$$

where $M_{c,i}$ is defined above. Similarly, the degree centrality of technological classes ($k_{i,0}$) is given by the number of cities that exhibit RTA in a particular class (ubiquity):

$$UBIQUITY = K_{i,0} = \sum_c M_{c,i} \quad (2)$$

The knowledge complexity index combines information on both the 2-mode degree distribution of cities (diversity) and the 2-mode degree distribution of the technologies produced (ubiquity). We follow Hidalgo and Hausmann (2009) and sequentially combine the diversity of cities and ubiquity of technological classes computing simultaneously the following 2 equations over a series of n iterations:

$$KCI_{cities} = K_{c,n} = \frac{1}{K_{c,0}} \sum_i M_{c,i} K_{i,n-1} \quad (3)$$

$$KCI_{tech} = K_{i,n} = \frac{1}{K_{i,0}} \sum_i M_{c,i} K_{c,n-1} \quad (4)$$

To provide some further interpretation of this method, in a second iteration, for $n = 1$, $K_{c,1}$ in equation (3) represents the average ubiquity of the technologies in which city *c* has RTA. In similar fashion, $K_{i,1}$ in equation (4) measures the average diversity of cities that have RTA in technology *i*. In the next iteration, for $n = 2$, $K_{c,2}$ captures the average diversity of cities that have export baskets similar to city *c*, and $K_{i,2}$ reveals the average ubiquity of the technologies developed in cities that have RTA in technology class i. Each additional step in KCI_{cities} yields a finer-grained estimate of the knowledge complexity of a city using information on the complexity of the technologies in which the city exhibits RTA. Each additional step in KCI_{tech} provides a finer-grained estimate of the knowledge complexity of a technology using information on the complexity of cities that have RTA in that technology. While higher order iterations in this technique become progressively more difficult to define, the method of reflections provides more and more precise measures of the KCI of cities and technologies, as noise and size effects are eliminated. The iterations are stopped when the ranking of cities and technologies is stable from one step to another (i.e. no further information can be extracted from the structure of the city-tech network). The KCI of cities presented in this paper is based on $n = 20$ iterations[4].

A reformulation of the 'method of reflections' as a fixed-point theorem based on Markov chain analysis[5] is provided by Caldarelli et al. (2012). Note that application of this alternative methodology does not substantially alter our results. New metrics derived from this mathematical reformulation are outlined in Tacchella et al. (2012).

5. The geography and evolution of complex knowledge

In this section, we present results of the structural analysis of the city-tech knowledge network with a particular focus on cities[6]. Some of the results are displayed for the 5-year period 2000-2004, though the patterns we describe are robust across all periods of observation.

An essential statistical indicator of global network structure is the degree distribution of nodes. As specified above, the degree of cities in the 2-mode network is simply given by the number of technologies in which a city has a relative technological advantage ($k_{c,0}$). The degree distribution gives the fraction of cities in the *city-tech knowledge network* with a given degree k. Figure 3 plots the cumulative degree distribution of cities for the period 2000-2004 and fits exponential (in black), power law (in red) and truncated power law (in light grey) functions to the data[7]. As it is often the case in 2-mode networks, the distribution is characterized by a power law

[4] The correlation between $K_{c,18}$ and $K_{c,20}$ is 0.99.

[5] We would like to thank Bernhard Truffer for pointing out the limitation of the method of reflections and suggesting an alternative algebraic solution. Vanessa Bouaroudj helped us write the R code that implements this mathematical reformulation.

[6] Following this approach, we could analyze the complexity of technological classes, but this is beyond the scope of the present paper.

[7] This figure has been plotted using the "bipartite" package, part of the R environment for statistical computing and graphics.

(β=0.48; R^2=0.80), with a better fit of the truncated power law (β=-0.37; R^2=0.99). This feature indicates a scale-free network (Barabási and Albert, 1999), in which only a few cities have RTA in many technological classes (hubs), while most cities have relatively low diversity scores. The MSAs with a high RTA tend to be large, such as Chicago, Los Angeles and Miami. However, some cities have a relatively low 2-mode degree despite a high patenting rate, such as San José, San Francisco and Boston. This indicates specialization of knowledge production in a small number of technology classes. The degree distribution of technologies, i.e. their ubiquity, follows a similar scale-free pattern characterized by a power law (β=0.34; R^2=0.76).

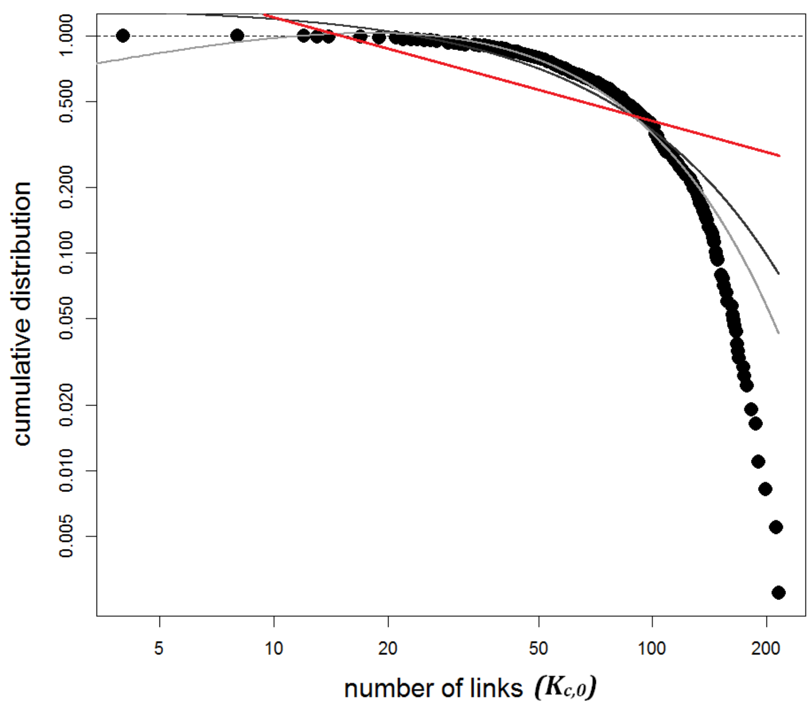


Figure 3. 2-mode degree distribution of cities (diversity) 2000-2004

In the next step of our structural analysis, we follow the approach of Hidalgo and Hausmann (2009) and analyze the relationship between (1) the diversity of the technologies produced by a city and (2) the average ubiquity of these technologies. In Figure 4, we plot the diversity of cities $K_{c,0}$ against the average ubiquity of technologies they produce $K_{c,1}$ for the period 2000-2004[8]. A high value of $K_{c,0}$ means that the technological structure of the city is highly diversified, while a low value of $K_{c,1}$ means that the city produces sophisticated, or non-ubiquitous, technologies on

[8] The relationship represented in Figure 4 holds for other time periods.

average. We find a strong negative relationship between these two indicators. This indicates that there is a strong tendency for cities with a more diversified technological structure to produce more exclusive (i.e. less ubiquitous) technologies.

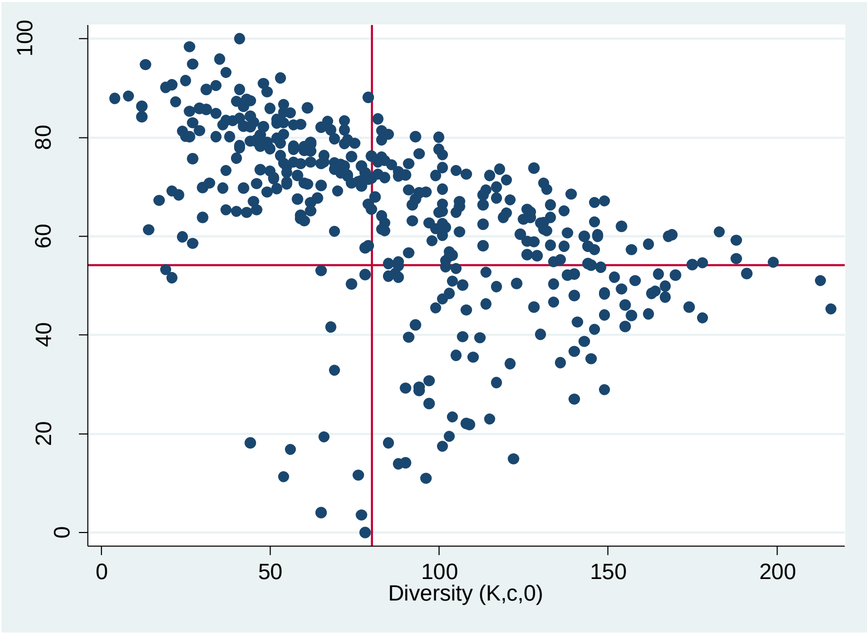
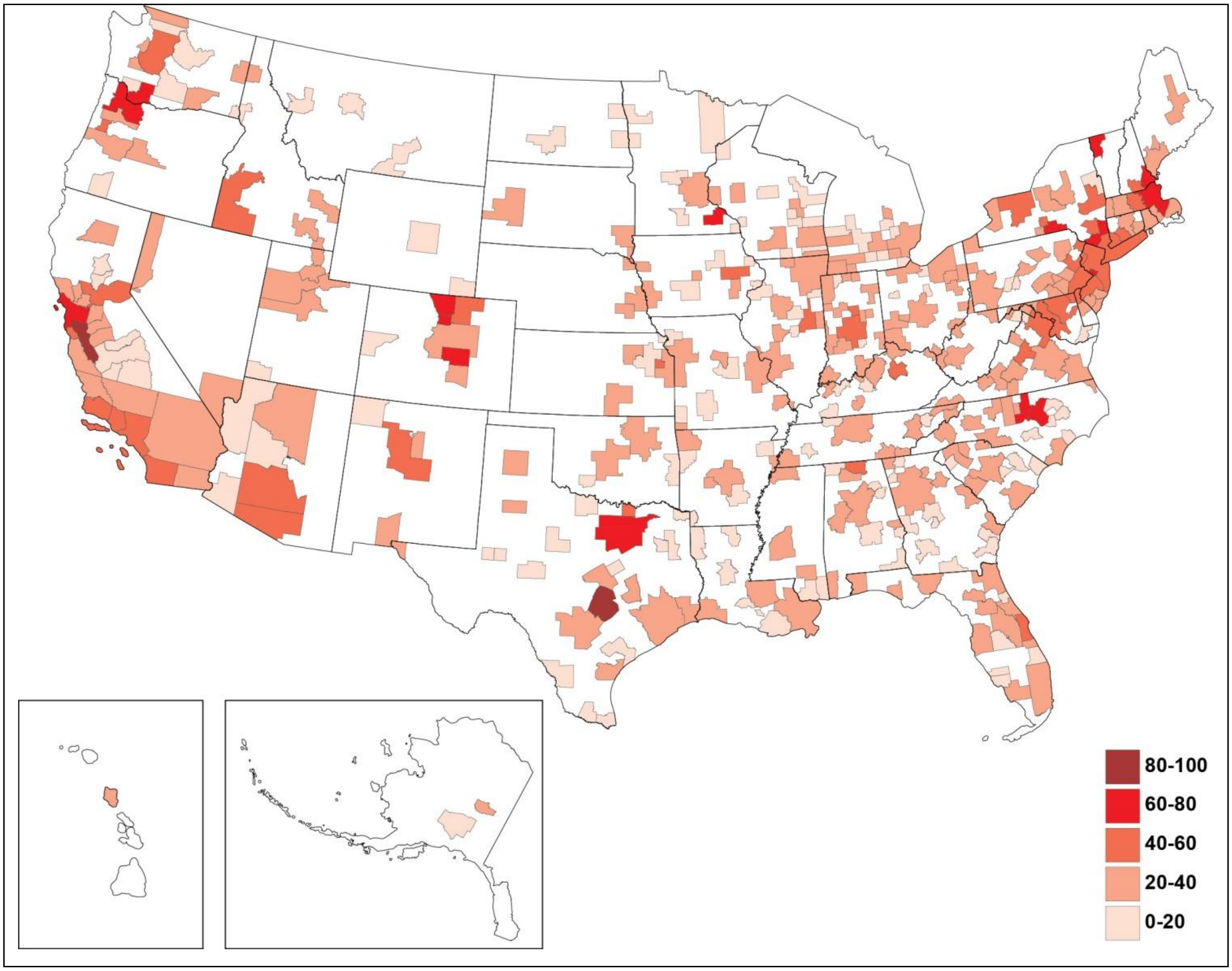


Figure 4. Diversity and average ubiquity of technologies produced (2000-2004)

Plotting the mean of diversity (vertical line) and the mean of average ubiquity (horizontal line) we divide the graph in four quadrants. In the bottom left quadrant we find cities that have RTA in relatively few technological classes that are non-ubiquitous. These are cities such as San José, Austin and Poughkeepsie. In the top left quadrant of the graph we also find cities that have RTA in only a few technological classes, but these cities (such as Anchorage, Springfield and Carson City) produce relatively ubiquitous technologies. On the right-hand side of the graph, we find larger, more diversified cities, producing sophisticated technologies in the bottom quadrant (Chicago, Los Angeles and New York for example), and cities producing more ubiquitous technologies in the upper quadrant (Columbia, Oklahoma City, New Orleans).

Looking at the diversity and average ubiquity of technologies produced provide interesting preliminary insights on the distribution of knowledge production in the United States that cannot be captured by simply counting aggregate knowledge outputs such as patent applications. These indicators, however, are based only on a small fraction of the entire structure of the city-tech knowledge network. Exploiting all information in this network, following the methodology of

Hidalgo and Hausmann (2009), we are able to characterize the complexity of the (technological) knowledge structure of the 366 Metropolitan Statistical Areas of the United States.



Notes: based on 20 iterations of the KCI.

Figure 5. Technological knowledge complexity in U.S. cities (average all years)

The knowledge complexity index for U.S. cities is quite heterogeneous as Figure 5 reveals. Knowledge complexity, averaged over our study period, is relatively high in San Jose, Austin, Poughkeepsie, San Francisco and Boston. These metropolitan areas tend to develop a number of technologies that can only be replicated in a small number of other U.S. cities (KCI > 70). Knowledge production is of moderately high complexity (60 < KCI < 70) in Rochester (MN), Burlington (VT), Trenton-Ewing, the Research Triangle cities of North Carolina, Colorado Springs, Fort Collins and Boulder, Binghampton, Dallas-Fort Worth and Portland (OR). The cities rounding out the top 10% in terms of average knowledge complexity since 1980 include Washington DC and New York City, Boise, Corvallis, Santa Cruz, Seattle, Phoenix and Tucson, Albany, Ithaca, Kingston and Rochester (NY), Manchester, Greeley, Worcester and Philadelphia.

Table 1 shows the KCI ranking of the 100 largest U.S. metropolitan areas in terms of employment (the KCI of the 366 MSAs is provided in Table A.1 of the Appendix). Interestingly, cities with the most complex technological knowledge structure are not necessarily the ones with the highest rates of patenting. Washington, for instance, ranks #134 in terms of patents per employee, but ranks #17 in terms of KCI. Thus, even though Washington produces relatively few patents, those patents tend to be concentrated in relatively sophisticated technological classes. A similar situation applies to cities such as Durham and Dallas. Conversely, we find at the bottom of the KCI ranking cities that tend to produce relatively large volumes of relatively ubiquitous technological knowledge (low KCI, approaching 0), such as Anchorage, Grand Rapids or Des Moines. One of the most striking results when we look at the drop in terms of ranking between KCI and the general rate of patenting is Detroit. Detroit ranks #45 in terms of the number of patents per employee over the period 1975-2010, but only #150 in terms of the knowledge complexity index. This indicates that Detroit is producing a substantial number of patents that could easily be produced by other cities. A similar portrait emerges of many other "rust-belt" cities like Akron, Minneapolis, Milwaukee and Toledo. These results suggest that looking at knowledge quality as well as knowledge quantity provides a somewhat different picture of the distribution of knowledge production in the United States.

MSA	**State**	**KCI**	**Rank (KCI)**	**Rank (patents)**	**MSA**	**State**	**KCI**	**Rank (KCI)**	**Rank (patents)**
San Jose-Sunnyvale-Santa Clara	CA	99	1	1	Providence-New Bedford-Fall River	RI	33	94	113
Austin-Round Rock-San Marcos	TX	94	2	10	Orlando-Kissimmee-Sanford	FL	32	96	264
Poughkeepsie-Newburgh-Middletown	NY	78	3	12	Denver-Aurora-Broomfield	CO	32	97	119
San Francisco-Oakland-Fremont	CA	77	4	19	Akron	OH	32	100	27
Boston-Cambridge-Quincy	MA	73	5	31	San Antonio-New Braunfels	TX	31	103	233
Trenton-Ewing	NJ	65	8	5	Columbus	OH	31	106	126
Colorado Springs	CO	64	9	75	El Paso	TX	31	107	317
Portland-Vancouver-Hillsboro	OR	64	10	47	Buffalo-Niagara Falls	NY	31	108	93
Raleigh-Cary	NC	62	11	25	Honolulu	HI	31	109	325
Dallas-Fort Worth-Arlington	TX	61	13	82	Portland-South Portland-Biddeford	ME	30	111	172
Durham-Chapel Hill	NC	60	16	44	Harrisburg-Carlisle	PA	30	114	118
Washington-Arlington-	DC	58	17	134	Springfield	MA	29	115	139

Alexandria									
New York-Northern New Jersey-Long Island	NY	58	18	80	Tampa-St. Petersburg-Clearwater	FL	29	117	178
San Diego-Carlsbad-San Marcos	CA	58	19	29	Columbia	SC	29	123	253
Rochester	NY	57	20	4	Richmond	VA	29	124	179
Boise City-Nampa	ID	57	21	3	Virginia Beach-Norfolk-Newport News	VA	28	128	277
Seattle-Tacoma-Bellevue	WA	56	24	51	Bakersfield-Delano	CA	28	129	173
Albany-Schenectady-Troy	NY	53	26	17	Kansas City	MO	28	131	196
Phoenix-Mesa-Glendale	AZ	52	28	66	Milwaukee-Waukesha-West Allis	WI	27	135	83
Worcester	MA	50	31	41	Charlotte-Gastonia-Rock Hill	NC	27	141	183
Philadelphia-Camden-Wilmington	PA	50	32	62	New Orleans-Metairie-Kenner	LA	27	142	243
Tucson	AZ	50	33	61	Greensboro-High Point	NC	26	145	198
Oxnard-Thousand Oaks-Ventura	CA	46	39	22	Detroit-Warren-Livonia	MI	26	150	45
Bridgeport-Stamford-Norwalk	CT	45	40	16	Riverside-San Bernardino-Ontario	CA	26	151	167
Albuquerque	NM	45	41	111	Jacksonville	FL	26	152	270
Allentown-Bethlehem-Easton	PA	44	42	42	Lansing-East Lansing	MI	26	153	181
Indianapolis-Carmel	IN	42	44	85	Scranton-Wilkes-Barre	PA	26	156	249
Sacramento-Arden-Arcade-Roseville	CA	42	45	146	Birmingham-Hoover	AL	26	157	298
Los Angeles-Long Beach-Santa Ana	CA	42	46	76	Toledo	OH	25	159	91
New Haven-Milford	CT	42	48	37	Las Vegas-Paradise	NV	25	160	247
Lexington-Fayette	KY	41	50	101	Oklahoma City	OK	25	161	211
Baltimore-Towson	MD	40	52	128	Charleston-North Charleston-Summerville	SC	25	164	240
Houston-Sugar Land-Baytown	TX	40	53	53	Fort Wayne	IN	25	172	92
Miami-Fort Lauderdale-Pompano Beach	FL	39	56	136	Wichita	KS	25	173	187
Ann Arbor	MI	38	61	13	Memphis	TN	24	176	245
Knoxville	TN	37	64	109	Nashville-Davidson-Murfreesboro-Franklin	TN	24	179	259
Syracuse	NY	37	66	96	Greenville-Mauldin-Easley	SC	24	181	162
Pittsburgh	PA	36	68	64	Little Rock-North Little Rock-Conway	AR	24	182	309
Minneapolis-St. Paul-Bloomington	MN	36	70	30	Winston-Salem	NC	23	184	144
Salt Lake City	UT	35	73	89	Omaha-Council Bluffs	NE	22	198	257
Chicago-Joliet-Naperville	IL	35	74	72	North Port-Bradenton-Sarasota	FL	22	205	135
Atlanta-Sandy Springs-Marietta	GA	34	79	143	Tulsa	OK	22	207	122
Madison	WI	34	81	69	Louisville-Jefferson County	KY	22	209	197
Lancaster	PA	34	83	33	Augusta-Richmond County	GA	22	213	282
Hartford-West Hartford-East Hartford	CT	34	85	54	Jackson	MS	21	220	341
Dayton	OH	34	88	70	Youngstown-Warren-Boardman	OH	21	228	200
Baton Rouge	LA	33	89	110	Chattanooga	TN	20	234	226
Cincinnati-Middletown	OH	33	90	55	Des Moines-West Des Moines	IA	19	254	155
St. Louis	MO	33	91	124	Grand Rapids-Wyoming	MI	19	256	138
Cleveland-Elyria-Mentor	OH	33	92	73	Fresno	CA	17	296	308

Table 1. Knowledge complexity index of the 100 largest MSAs (1975-2010)

So far, we have presented the knowledge complexity index for the entire 1975-2004 period. But of course, cities are continuously changing their technological portfolio as they diversify into new technological classes (Colombelli et al., 2012 Essleztbichler, 2013; Rigby 2013; Boschma et al., 2014). As a result the complexity of technological knowledge is also evolving over time. Figure 6 plots a standardized version of the knowledge complexity index of cities for the periods 1975-1984 and 1995-2004 against each other, so we can observe if the KCI of individual cities has improved/declined over time.

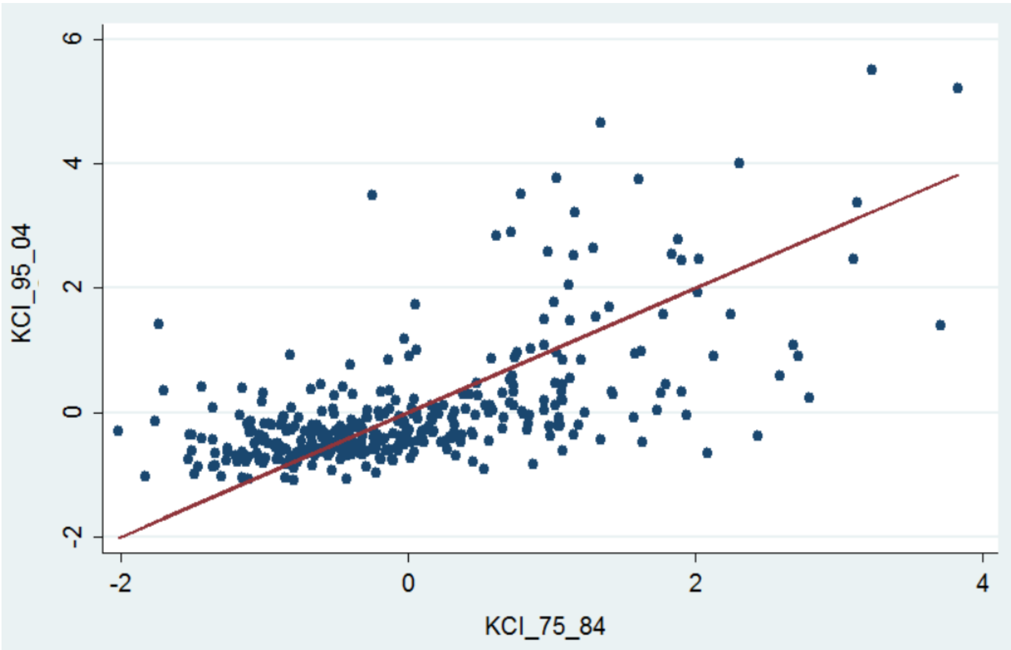


Figure 6. Evolution of the knowledge complexity index over time

Cities that are located above the 45° red line have improved the complexity of their technological knowledge structures, while the cities located below the line have experienced a decline in their KCI (relatively to other cities). Boise, Seattle and Austin have registered strong gains in the complexity of their knowledge structure. At the same time, cities such as Philadelphia, Baton Rouge and New Orleans now have a significantly lower KCI than in the mid-1970s. What is interesting about this graph is that, for most cities, the knowledge complexity index is relative stable over time. This provides some indication of strong path dependence in the evolution of technological structure.

If we focus exclusively on newly added technological classes, a similar pattern emerges. The average complexity of the newly added technological classes in a city, from one period to the next, is strongly correlated to lagged KCI. These data support the arguments about knowledge development being a cumulative process of recombining existing skills and competencies.

The geography of shifts in knowledge complexity by metropolitan area is shown more clearly in Figure 7. The red shading in this figure indicates those metropolitan areas that have experienced increases in KCI from the period 1975-1984 to 1995-2004, while the blue shading indicates declining KCI. The legend reports changes in normalized values of KCI over the two periods. Figure 7 shows the general decline in the complexity of knowledge produced across much of the snow-belt of the United States, along with some cities from the South. The most significant gains in knowledge complexity are recorded by Boise and Sioux City over the 30-year period examined. Relatively strong gains in KCI are also registered by Portland and Seattle, Fort Collins and Colorado Springs, Austin, Santa Cruz and Merced, by Burlington (VT) and Fairbanks.

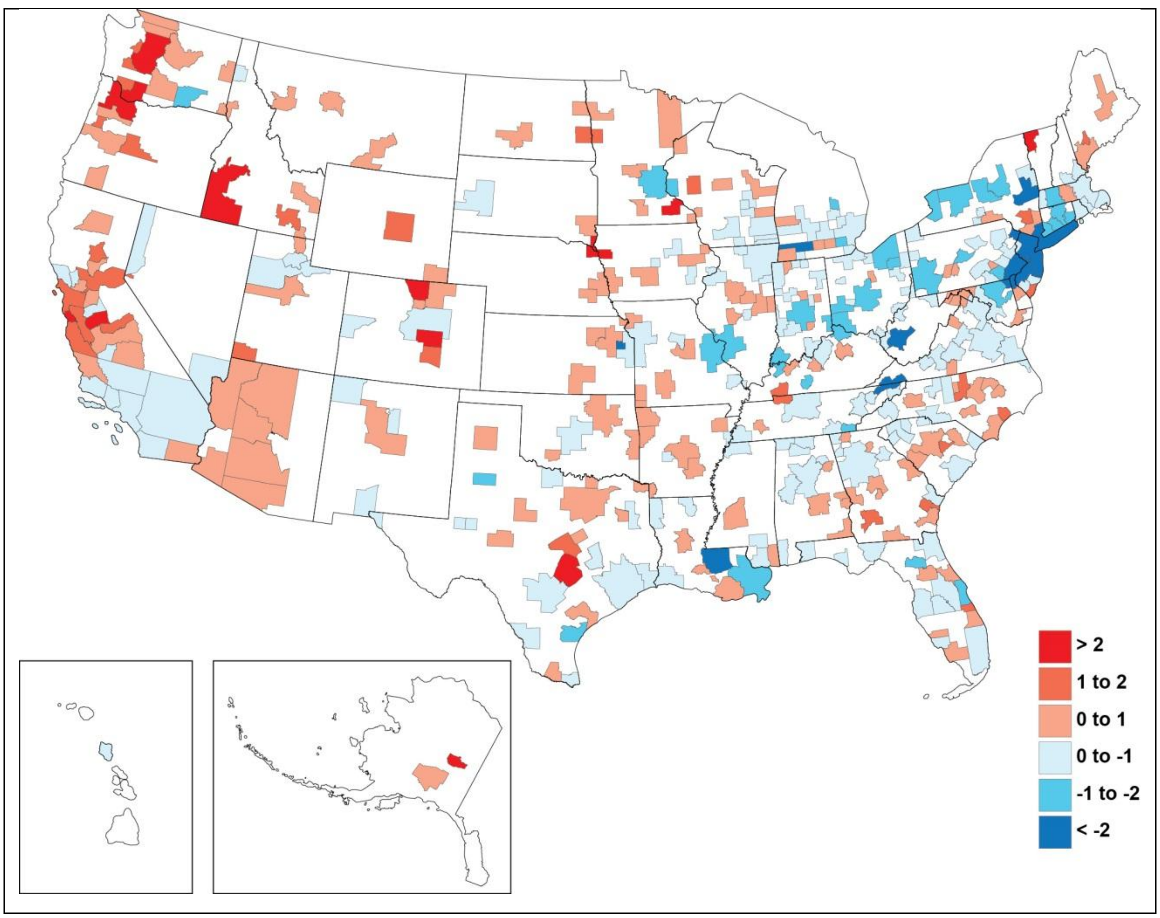


Figure 7: Changes in KCI 1975-1984 to 1995-2004

The largest declines in KCI are registered by Kalamazoo, Charleston (WV), Philadelphia and Trenton-Ewing, Kingsport-Bristol (TN-VA), Albany and New York City, Lawrence and Baton Rouge.

6. Conclusion and discussion

Knowledge is an increasingly critical dimension of competitive advantage. While past work has explored the geography of patenting, this work largely treats individual patents as homogeneous, assuming that each patent adds only as much technological potential to a region's economy as the next. However, not all patents hold the same value. Recent work has shown how the knowledge cores of countries and regions might be differentiated using patent data and measures of the technological relatedness between patents in different classes. In this paper we extend the method of reflections of Hidalgo and Hausmann (2009) to generate measures of the knowledge complexity of patents generated across U.S. metropolitan areas since 1975.

Our analysis reveals that there are wide geographical variations in knowledge complexity, with only a few metro regions producing the most complex new technologies at any one time. There is considerable rank stability in the positions of most cities in terms of the complexity of knowledge embodied in patents across the five-year periods that we examined. However, many snow-belt cities, and cities in the South, have witnessed a slow decline in the complexity of the knowledge that they are producing. Across a number of metropolitan areas in the West, and a few selected cities in the East, the complexity of knowledge produced has generally increased over the last thirty years or so. These shifts in knowledge complexity are connected to cross-sectional differences in gross metropolitan product per capita and thus to average income levels, as well as to rates of economic growth.

The cities producing the most complex new technologies appear to be capturing a growing share of all new knowledge generated within the United States. There is evidence that networks of inventors across the U.S. are being reconfigured and growing in density around the complex knowledge hubs that we identify. Of course, not all knowledge is spatially sticky. Low complexity, more routinized, forms of knowledge are still being produced across many U.S. metropolitan areas. However, the development of low complexity knowledge is increasingly footloose and provides an insecure foundation of competitive advantage. Much more work remains to be done on these issues and what they imply for the future of U.S. cities.

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Appendix

MSA	State	KCI	Rank (KCI)	Rank (patents)
San Jose-Sunnyvale-Santa Clara	CA	99	1	1
Austin-Round Rock-San Marcos	TX	94	2	10
Poughkeepsie-Newburgh-Middletown	NY	78	3	12
San Francisco-Oakland-Fremont	CA	77	4	19
Boston-Cambridge-Quincy	MA	73	5	31
Rochester	MN	73	6	11
Burlington-South Burlington	VT	68	7	8
Trenton-Ewing	NJ	65	8	5
Colorado Springs	CO	64	9	75
Portland-Vancouver-Hillsboro	OR	64	10	47
Raleigh-Cary	NC	62	11	25
Binghamton	NY	62	12	24
Dallas-Fort Worth-Arlington	TX	61	13	82
Fort Collins-Loveland	CO	61	14	7
Boulder	CO	61	15	6
Durham-Chapel Hill	NC	60	16	44
Washington-Arlington-Alexandria	DC	58	17	134
New York-Northern New Jersey-Long Island	NY	58	18	80
San Diego-Carlsbad-San Marcos	CA	58	19	29
Rochester	NY	57	20	4
Boise City-Nampa	ID	57	21	3
Kingston	NY	57	22	43
Santa Cruz-Watsonville	CA	57	23	14
Seattle-Tacoma-Bellevue	WA	56	24	51
Corvallis	OR	55	25	2
Albany-Schenectady-Troy	NY	53	26	17
Manchester-Nashua	NH	53	27	28
Phoenix-Mesa-Glendale	AZ	52	28	66
Ithaca	NY	51	29	15
Greeley	CO	50	30	9
Worcester	MA	50	31	41
Philadelphia-Camden-Wilmington	PA	50	32	62
Tucson	AZ	50	33	61

Palm Bay-Melbourne-Titusville	FL	48	34	18
Cedar Rapids	IA	48	35	56
Huntsville	AL	47	36	71
Kokomo	IN	47	37	50
Champaign-Urbana	IL	46	38	98
Oxnard-Thousand Oaks-Ventura	CA	46	39	22
Bridgeport-Stamford-Norwalk	CT	45	40	16
Albuquerque	NM	45	41	111
Allentown-Bethlehem-Easton	PA	44	42	42
Santa Barbara-Santa Maria-Goleta	CA	43	43	35
Indianapolis-Carmel	IN	42	44	85
Sacramento-Arden-Arcade-Roseville	CA	42	45	146
Los Angeles-Long Beach-Santa Ana	CA	42	46	76
Sherman-Denison	TX	42	47	140
New Haven-Milford	CT	42	48	37
Charlottesville	VA	41	49	104
Lexington-Fayette	KY	41	50	101
Lawrence	KS	40	51	107
Baltimore-Towson	MD	40	52	128
Houston-Sugar Land-Baytown	TX	40	53	53
Norwich-New London	CT	40	54	34
State College	PA	40	55	86
Miami-Fort Lauderdale-Pompano Beach	FL	39	56	136
Provo-Orem	UT	39	57	68
Gainesville	FL	39	58	65
Eau Claire	WI	39	59	154
Blacksburg-Christiansburg-Radford	VA	38	60	84
Ann Arbor	MI	38	61	13
Santa Rosa-Petaluma	CA	38	62	63
Iowa City	IA	37	63	114
Knoxville	TN	37	64	109
College Station-Bryan	TX	37	65	117
Syracuse	NY	37	66	96
Lynchburg	VA	37	67	130
Pittsburgh	PA	36	68	64
Pocatello	ID	36	69	209
Minneapolis-St. Paul-Bloomington	MN	36	70	30
Kennewick-Pasco-Richland	WA	36	71	60
Ames	IA	36	72	23
Salt Lake City	UT	35	73	89
Chicago-Joliet-Naperville	IL	35	74	72
Las Cruces	NM	35	75	224
Utica-Rome	NY	35	76	223
Santa Fe	NM	35	77	120

Kalamazoo-Portage	MI	34	78	32
Atlanta-Sandy Springs-Marietta	GA	34	79	143
Charleston	WV	34	80	150
Madison	WI	34	81	69
Salinas	CA	34	82	207
Lancaster	PA	34	83	33
Kingsport-Bristol-Bristol	TN	34	84	59
Hartford-West Hartford-East Hartford	CT	34	85	54
Lafayette	IN	34	86	57
Ocean City	NJ	34	87	279
Dayton	OH	34	88	70
Baton Rouge	LA	33	89	110
Cincinnati-Middletown	OH	33	90	55
St. Louis	MO	33	91	124
Cleveland-Elyria-Mentor	OH	33	92	73
Elmira	NY	33	93	21
Providence-New Bedford-Fall River	RI	33	94	113
Johnson City	TN	32	95	193
Orlando-Kissimmee-Sanford	FL	32	96	264
Denver-Aurora-Broomfield	CO	32	97	119
Roanoke	VA	32	98	177
Tuscaloosa	AL	32	99	321
Akron	OH	32	100	27
Reading	PA	32	101	125
Pittsfield	MA	31	102	81
San Antonio-New Braunfels	TX	31	103	233
Ogden-Clearfield	UT	31	104	87
Athens-Clarke County	GA	31	105	149
Columbus	OH	31	106	126
El Paso	TX	31	107	317
Buffalo-Niagara Falls	NY	31	108	93
Honolulu	HI	31	109	325
Hattiesburg	MS	30	110	296
Portland-South Portland-Biddeford	ME	30	111	172
Bloomington	IN	30	112	170
Reno-Sparks	NV	30	113	94
Harrisburg-Carlisle	PA	30	114	118
Springfield	MA	29	115	139
El Centro	CA	29	116	215
Tampa-St. Petersburg-Clearwater	FL	29	117	178
Bremerton-Silverdale	WA	29	118	157
Crestview-Fort Walton Beach-Destin	FL	29	119	314
Lebanon	PA	29	120	158
Sebastian-Vero Beach	FL	29	121	156

Morgantown	WV	29	122	165
Columbia	SC	29	123	253
Richmond	VA	29	124	179
Lubbock	TX	29	125	230
Stockton	CA	28	126	244
Spokane	WA	28	127	202
Virginia Beach-Norfolk-Newport News	VA	28	128	277
Bakersfield-Delano	CA	28	129	173
Evansville	IN	28	130	116
Kansas City	MO	28	131	196
Olympia	WA	28	132	238
Idaho Falls	ID	27	133	67
Decatur	AL	27	134	222
Milwaukee-Waukesha-West Allis	WI	27	135	83
South Bend-Mishawaka	IN	27	136	78
Killeen-Temple-Fort Hood	TX	27	137	345
Harrisonburg	VA	27	138	327
Eugene-Springfield	OR	27	139	127
Modesto	CA	27	140	201
Charlotte-Gastonia-Rock Hill	NC	27	141	183
New Orleans-Metairie-Kenner	LA	27	142	243
Columbia	MO	27	143	180
Cumberland	MD	26	144	312
Greensboro-High Point	NC	26	145	198
Bend	OR	26	146	95
Auburn-Opelika	AL	26	147	153
Barnstable Town	MA	26	148	108
Tallahassee	FL	26	149	261
Detroit-Warren-Livonia	MI	26	150	45
Riverside-San Bernardino-Ontario	CA	26	151	167
Jacksonville	FL	26	152	270
Lansing-East Lansing	MI	26	153	181
Salem	OR	26	154	219
Burlington	NC	26	155	218
Scranton-Wilkes-Barre	PA	26	156	249
Birmingham-Hoover	AL	26	157	298
Vallejo-Fairfield	CA	25	158	199
Toledo	OH	25	159	91
Las Vegas-Paradise	NV	25	160	247
Oklahoma City	OK	25	161	211
Danville	IL	25	162	248
York-Hanover	PA	25	163	115
Charleston-North Charleston-Summerville	SC	25	164	240
Manhattan	KS	25	165	254

Terre Haute	IN	25	166	225
Niles-Benton Harbor	MI	25	167	52
San Luis Obispo-Paso Robles	CA	25	168	106
Mobile	AL	25	169	295
Danville	VA	25	170	351
Napa	CA	25	171	164
Fort Wayne	IN	25	172	92
Wichita	KS	25	173	187
Spartanburg	SC	25	174	36
Asheville	NC	24	175	133
Memphis	TN	24	176	245
Panama City-Lynn Haven-Panama City Beach	FL	24	177	169
Logan	UT	24	178	46
Nashville-Davidson-Murfreesboro-Franklin	TN	24	179	259
Deltona-Daytona Beach-Ormond Beach	FL	24	180	190
Greenville-Mauldin-Easley	SC	24	181	162
Little Rock-North Little Rock-Conway	AR	24	182	309
Lincoln	NE	24	183	166
Winston-Salem	NC	23	184	144
Springfield	OH	23	185	189
Elkhart-Goshen	IN	23	186	103
Wheeling	WV	23	187	297
Carson City	NV	23	188	77
Wilmington	NC	23	189	176
Coeur d'Alene	ID	23	190	186
Appleton	WI	23	191	38
Bellingham	WA	23	192	105
Corpus Christi	TX	23	193.5	242
Bowling Green	KY	23	193.5	311
Canton-Massillon	OH	23	195	100
Williamsport	PA	23	196	102
Pueblo	CO	23	197	322
Omaha-Council Bluffs	NE	22	198	257
Hagerstown-Martinsburg	MD	22	199	301
Bay City	MI	22	200	79
Holland-Grand Haven	MI	22	201	20
Dover	DE	22	202	287
Peoria	IL	22	203	39
Rockford	IL	22	204	58
North Port-Bradenton-Sarasota	FL	22	205	135
Hot Springs	AR	22	206	310
Tulsa	OK	22	207	122
Bangor	ME	22	208	340
Louisville-Jefferson County	KY	22	209	197

Savannah	GA	22	210	323
Beaumont-Port Arthur	TX	22	211	237
Hickory-Lenoir-Morganton	NC	22	212	204
Augusta-Richmond County	GA	22	213	282
Fairbanks	AK	22	214	331
Cleveland	TN	22	215.5	121
Huntington-Ashland	WV	22	215.5	251
Erie	PA	22	217	90
Parkersburg-Marietta-Vienna	WV	22	218	141
Anderson	IN	21	219	123
Jackson	MS	21	220	341
Ocala	FL	21	221	235
Gainesville	GA	21	222	205
Fayetteville-Springdale-Rogers	AR	21	223	252
Muncie	IN	21	224	231
Lewiston-Auburn	ME	21	225	294
Sioux Falls	SD	21	226	281
Vineland-Millville-Bridgeton	NJ	21	227	292
Youngstown-Warren-Boardman	OH	21	228	200
Pensacola-Ferry Pass-Brent	FL	21	229	234
Cape Girardeau-Jackson	MO	21	230	246
Rapid City	SD	21	231	305
Flagstaff	AZ	20	232	151
Springfield	IL	20	233	250
Chattanooga	TN	20	234	226
Amarillo	TX	20	235	303
Sioux City	IA	20	236	210
Atlantic City-Hammonton	NJ	20	237	320
Naples-Marco Island	FL	20	238	148
Lake Charles	LA	20	239	289
Redding	CA	20	240	216
Merced	CA	20	241	343
Glens Falls	NY	20	242	99
Gadsden	AL	20	243	354
Anderson	SC	20	244	174
Lakeland-Winter Haven	FL	20	245	236
La Crosse	WI	20	246	192
Anniston-Oxford	AL	20	247	356
Sumter	SC	20	248	352
Racine	WI	20	249	40
Port St. Lucie	FL	19	250	112
Prescott	AZ	19	251	147
Kankakee-Bradley	IL	19	252	213
Waco	TX	19	253	313

Des Moines-West Des Moines	IA	19	254	155
Mansfield	OH	19	255	229
Grand Rapids-Wyoming	MI	19	256	138
Lima	OH	19	257	293
Greenville	NC	19	258	263
Chico	CA	19	259	239
Davenport-Moline-Rock Island	IA	19	260	160
Medford	OR	19	261	195
Columbus	GA	19	262	350
Altoona	PA	19	263	307
Montgomery	AL	19	264	339
Flint	MI	19	265	131
Topeka	KS	19	266	336
Grand Junction	CO	19	267	185
Florence	SC	19	268	206
Myrtle Beach-North Myrtle Beach-Conway	SC	19	269	329
Decatur	IL	19	270	145
Rome	GA	19	271	260
Mankato-North Mankato	MN	19	272	194
Janesville	WI	19	273	132
Macon	GA	19	274	288
Florence-Muscle Shoals	AL	19	275	203
Monroe	MI	18	276	49
Missoula	MT	18	277	232
Longview	TX	18	278	175
Cape Coral-Fort Myers	FL	18	279	191
Midland	TX	18	280	171
Saginaw-Saginaw Township North	MI	18	281	129
Cheyenne	WY	18	282	318
Dalton	GA	18	283	275
Morristown	TN	18	284	255
Fond du Lac	WI	18	285	88
Fargo	ND	18	286	212
Steubenville-Weirton	OH	18	287	274
Joplin	MO	18	288	184
Oshkosh-Neenah	WI	18	289	26
Valdosta	GA	18	290	353
Gulfport-Biloxi	MS	17	291	316
Elizabethtown	KY	17	292	334
Dothan	AL	17	293	347
Brownsville-Harlingen	TX	17	294	360
Yuma	AZ	17	295	332
Fresno	CA	17	296	308
Owensboro	KY	17	297	283

Bloomington-Normal	IL	17	298	241
Columbus	IN	17	299	48
Visalia-Porterville	CA	17	300	324
Green Bay	WI	17	301	208
Duluth	MN	17	302	285
Yuba City	CA	16	303	268
Longview	WA	16	304	221
Punta Gorda	FL	16	305	168
Lewiston	ID	16	306	315
Muskegon-Norton Shores	MI	16	307	137
McAllen-Edinburg-Mission	TX	16	308	363
Springfield	MO	16	309	217
Johnstown	PA	16	310	306
Wichita Falls	TX	16	311	269
Lake Havasu City-Kingman	AZ	16	312	265
Jefferson City	MO	16	313	338
Goldsboro	NC	16	314	358
Odessa	TX	16	315	188
St. George	UT	16	316	220
Anchorage	AK	16	317	280
St. Cloud	MN	16	318	262
Lafayette	LA	16	319	161
Tyler	TX	15	320	258
Sandusky	OH	15	321	142
Fayetteville	NC	15	322	362
Grand Forks	ND	15	323	284
Lawton	OK	15	324	361
Jackson	MI	15	325	97
Sheboygan	WI	15	326	74
Hinesville-Fort Stewart	GA	15	327	366
Winchester	VA	15	328	266
Abilene	TX	15	329	337
Albany	GA	15	330	346
Shreveport-Bossier City	LA	15	331	302
Warner Robins	GA	15	332	355
Monroe	LA	15	333	256
Great Falls	MT	14	334	333
Madera-Chowchilla	CA	14	335	271
Wausau	WI	14	336	182
Pascagoula	MS	14	337	330
St. Joseph	MO	14	338	286
Laredo	TX	14	339	364
Pine Bluff	AR	13	340	357
Michigan City-La Porte	IN	13	341	159

Jonesboro	AR	13	342	304
Wenatchee-East Wenatchee	WA	13	343	326
Billings	MT	13	344	300
Farmington	NM	13	345	276
Battle Creek	MI	13	346	278
Salisbury	MD	13	347	290
Waterloo-Cedar Falls	IA	13	348	152
Victoria	TX	13	349	335
Yakima	WA	12	350	272
Fort Smith	AR	12	351	319
Palm Coast	FL	12	352	214
Rocky Mount	NC	12	353	349
Hanford-Corcoran	CA	12	354	359
Clarksville	TN	12	355	344
Mount Vernon-Anacortes	WA	12	356	273
Jacksonville	NC	12	357	365
Jackson	TN	11	358	227
Alexandria	LA	11	359	328
Brunswick	GA	11	360	299
Casper	WY	11	361	291
Dubuque	IA	10	362	163
San Angelo	TX	10	363	348
Houma-Bayou Cane-Thibodaux	LA	10	364	228
Bismarck	ND	9	365	267
Texarkana	TX	6	366	342

Note: This table indicates the KCI of all 366 metros). The KCI presented in column 3 is a rounded average of the KCI of cities over the 6 time periods. Column 4 gives the corresponding ranking while column 5 indicates the ranking of MSA based on the patents/employees ratio.

Table A.1. Knowledge complexity index of the 100 biggest U.S. cities