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The economic geography can be viewed as a large and growing network of interacting activities. This fundamental network structure and the large size of such systems makes complex networks an attractive model for its analysis. In this paper we propose the use of complex networks for geographical modeling and demonstrate how such an application can be combined with a cellular model to produce output that is consistent with large scale regularities such as power laws and fractality. Complex networks can provide a stringent framework for growth dynamic modeling where concepts from e.g. spatial interaction models and multiplicative growth models can be combined with the flexible representation of land and behavior found in cellular automata and agent-based models. In addition, there exists a large body of theory for the analysis of complex networks that have direct applications for urban geographic problems. The intended use of such models is twofold: i) to address the problem of how the empirically observed hierarchical structure of settlements can be explained as a stationary property of a stochastic evolutionary process rather than as equilibrium points in a dynamics, and, ii) to improve the prediction quality of applied urban modeling.

I. INTRODUCTION

One possible definition of geography is the study of uneven distributions in space and to capture various aspects of such distributions is the over-arching aim of models of urban dynamics. On a general level such distributions exhibit a number of statistical regularities, such as fractality or the presence of power laws and the hierarchical structure of human settlements (Andersson et al., 2003; Batty and Longley, 1994; Ioannides and Overman, 2002; Kaizoji, 2003; Rosen and Resnick, 1980; Soo, 2002). Such regularities can be viewed as important constraints on the output that can be accepted from models because they represent knowledge about the geography of land use in general. Present understanding of such regularities largely resides in a diverse set of abstract theoretical models and these are mostly static or equilibrium models or they are formulated on an aggregated scale such as cities(Pumain, 2000). These models are hard to unify and they are hard to garnish with additional details needed for application in scenario prediction. Creating urban growth models that are consistent with global regularities, that at the same time are open-ended for addition of the wealth of details necessary for application in e.g. scenario prediction must be considered an important research direction. The present paper represents an attempt to work in this direction by applying "complex networks" to urban growth models.

The best explanations for large scale regularities in urban systems are currently provided by multiplicative growth models based on the so-called Gibrat's law which states that the relative growth rates of cities is constant. Simon's model and Gabaix extension of this model, which solves the problem of infinitely slow convergence, are among the most widely cited such models(Gabaix, 1999; Simon, 1955). Although the Gabaix model essentially solves the problem of Zipf's Law for city sizes in a satisfying way it does have some unwanted properties that relates to its formulation on the macroscopic scale of cities. Skewed distributions appear on lower levels than that of cities and therefore it appears that an explanation on the city level is not fundamental. In the present as well as earlier papers we have demonstrated that scale-free distributions appear already on the level of land values per unit area(Andersson et al., 2005, 2003). This finding is confirmed also for Japan by Kaizoji (2003). Although crucial as criteria for admissability of models, the importance of large scale regularities in urban systems should however not be over-stated. In complex systems such as the urban system macroscopic states of this kind do not exhaust the properties of the system nearly as well as what is often the case for physical systems. In other words, a lot of important dynamics (in many cases emergent) takes place in the urban system that are not captured by large scale regularities and thus these regularities enter as constraints that may disqualify but certainly not qualify candidate mechanisms on lower levels. Models formulated on the macro level hide a lot of important dynamics and this has some implications. For example it is

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hard to relax critical assumptions on local growth rates in a realistic way in a macroscopic model. Such models do not allow us to properly understand when, why and in which ways urban systems conform to or deviate from these regularities. To do this, it appears reasonable that micro dynamics need to be considered explicitly. For instance, spatial interaction and a variety of often cited “forces”, e.g. agglomeration economies, economies of scale, immobile inputs and so on, act on levels below that of entire cities. Another implication is that macroscopic models can not be used for addressing equally important issues of internal structure of urban systems – something that apart from being of theoretical interest also is of critical importance for forecasting and scenario exploration models. Furthermore, cities are hard to define and measure and because of this a range of methods have been used when collecting data about rates of growth and populations. The problem of defining and measuring cities is discussed for example by Pumain (2004).

To be able to use the knowledge that stationary power laws stem from multiplicative growth in more detailed models we must be able to incorporate more detailed growth dynamics. Using a complex networks model mapped to a cellular space we may use multiplicative growth on a micro level and incorporate, in a flexible and open-ended manner, any number of sub models that capture forces such as transportation costs, external economies, economies of scale and agglomeration economies. Doing this we may investigate whether, in which way, under what circumstances such forces affect global regularities. We may also formulate richer models capable of reproducing the finer details of internal urban structure that also retain consistency with known regularity constraints such as power laws on a larger scale. It must be noted that this has not been done in the present paper where the focus is instead on large scale regularities.

The range of possible applications of complex networks in economic geography is wide and we will limit the discussion here to their use for urban growth modeling. The representation of land that we use here is highly similar to that of a cellular automaton, and many of the attractive properties that are associated with cellular models also apply here. However, the mathematical toolbox that applies to complex networks, and at the same time pertains to relevant issues for geographic modeling, is considerably more general. Furthermore the central importance of interactions in economic geography is reflected in the fundamental structure of complex networks. Edge growth and long-range correlations that may lead to fractal patterns also enters naturally. In that sense, complex networks models retain and extend the functional advantages of previous models and additionally brings a substantial battery of analytical and statistical methods to bear at the problem.

We will adhere to the following structure. First we discuss some properties of the economic geography as a complex dynamical system and arrive at a rationale for using complex networks to address some important questions pertaining to the generation of macrostructure from microscopic mechanisms. We then introduce a baseline model of complex evolving networks. Thereafter, we proceed to define a model of the proposed type for the purpose of simulating the geographical evolution of land values, clusters and patterns. We also present some results and interpretations of this model and discuss some of the problems and benefits with this formulation. Finally we discuss the application of complex networks to economic geography in a wider context. The basic assumptions underlying our model have also been used in previous papers by (Andersson et al., 2005, 2003). In this contribution, we elaborate on the model by introducing the concept of node fitness, and by discussing the methodology, model set-up, and model properties in the context of economic geography more widely.

II. A RATIONALE FOR USING COMPLEX NETWORKS FOR GEOGRAPHICAL MODELING

Economic geography is today often considered to be less about the “space of places”, where the success of places is attributed to characteristics of neighborhoods, and more about the “the space of flows” where places derive their success from their position in networks. General principles underlying the economic geography of networks have been applied to a variety of topics including trade networks, transportation networks, urban systems, ICT networks, knowledge networks, cultural networks)(Castells, 2000; Hohenberg and Lees, 1995; Moss and Townsend, 2000). Networks consist of interacting nodes and interactions between geographical locations are fundamental to present understanding of geographical systems of human origin, for example in spatial interaction models following Isard (1960). However, networks in economy and society are typically hard to analyze just by observing them. They are very large, involving thousands or millions of nodes and connections and rather than representing some schematic structure they represent almost arbitrary outcomes of a network growth dynamics. Furthermore, these systems generally have a high degree of inertia and change slowly over time which means that they do not lend themselves well to equilibrium analysis based on comparative statics.

The last few decades have seen a dramatic increase in availability of data and a correspondingly dramatic decrease in cost of memory storage and CPU cycles. These two important developments have opened the doors for models and methods of data analysis that were unthinkable only ten or twenty years ago. Complex networks, an area of study that originally stems from graph theory and statistical physics is an example of this type of models. This approach has been extensively explored over the last decade for studying the dynamics of large growing networks in a very wide range of areas, including the World Wide Web (Barabási et al., 2000), the Internet (Cohen et al., 2001, 2000; Pastor-Satorras et al., 2001), human sexual contacts (Liljeros et al., 2001), ecological networks (Camacho et al., 2002), cellular networks (Jeong et al., 2000; Ravasz et al., 2002), social networks (Girvan and Newman, 2002; Holme, 2003; Jin et al., 2001), phone calls (Abello et al., 1999; Aiello et al., 2000), citations (i Cancho and Solé, 2001; Redner, 1998) and protein folding (Scala et al., 2001). Common to these systems is the fact that they are networks that are very large realizations of an underlying growth dynamics. Due to the present availability of large geographical data sets and the inherent characteristics of the economic geography as a large growing network, complex networks appear to be an attractive baseline model.

III. A CONCEPTUAL MODEL THAT CAN BE EXTENDED TO MORE TOPICAL INCARNATIONS

As a very general observation, models of urban evolution aim to reproduce properties of urban systems. However, such properties must be viewed as existing at different levels of observation, and, for different purposes properties on different levels are of primary interest. For example, a power law distribution of city sizes is clearly a property of most urban systems, but it is not a property of any urban system in particular. Because of this, a model that only predicts this very general regularity is of little use for directing planners and policy-makers in making decisions about any particular region. In such cases we are more interested in properties of particular systems; systems identified for example by a boundary condition, empirically estimated parameters and ad hoc growth rules. We might in such cases for instance start from some configuration and calibrate a cellular automaton model to extrapolate this state into the future. Such a model will be less useful for addressing fundamental questions about urban evolution, but more useful for answering question about some specific region. For reasons that follow, models based on complex networks mapped to a cellular space may provide a new way of unifying general and special predictive power into the same model: i) a complex networks model may operate on the micro-mechanism level, the same level at which also cellular automata and agent-based models operate, and, ii) a complex networks model also allows for the incorporation of mechanisms that have been proposed through theoretical models of urban growth such as multiplicative growth and interaction concepts from spatial interaction models. Such a model can capture the network properties of the economy while providing room for flexibility in implementation similar to that found in cellular automata. Furthermore, the assumptions that have to be made in order to keep the model simple is of a different and potentially more innocuous kind than those made for economic models. This will be discussed more in detail later on in the paper.

Cellular automata models and agent based models are to be preferred if the objective is to faithfully represent complex spatial mechanisms, yet these approaches often suffer from limited mathematical tractability. In contrast, mapping the same system as a complex network means that we can start to apply the body of theory that pertains to complex network to problems in economic geography. It must be pointed out that the example demonstrated in the paper makes no claim on being the only or the uniquely correct way of making such a mapping. However, using this representation we verify a number of predictions that can be made directly from the fact that we are observing a growing network of a particular type. For example, the power law distribution of node degrees, which we interpret as land values, follows analytically from the preferential attachment mechanism that applies to a wide range of networks. Furthermore, we introduce the concept of "endogenous node fitness", the local rate of growth, in an economic geography setting. This concept is extended from a similar concept for general networks which is also found in the literature (Bianconi and Barabási, 2001; Ergün and Rodgers, 2002). The application of general tools of analysis to the economic geography viewed as a complex network can with all likelihood be carried much further than the analysis made here and this will be discussed more in depth later in this article. For example, none of our analysis presently concerns the historical path dependence of many urban phenomena. In addition, because a complex network model simulates the evolution of interactions between nodes, several analytic concepts from spatial interaction modeling might also apply although this has not been explored thus far.

The term "model" bears somewhat different connotations for different readers. Whereas "model" to a planner might indicate a tool for prediction of specific scenarios, a "model" to more theory-minded person would suggest a more general and conceptual tool for understanding the connection between mechanisms and phenomena. We will refer to these ends of the spectrum of models as *topical* and *conceptual* types of models. E.g. Simon's model would be a more conceptual model while a calibrated cellular automaton would be a more topical model. This distinction is important because models along the entire spectrum are being developed and explored in the context on urban growth and this might influence the expectations of individual readers.

Conceptual models are constructed to address fundamental questions about systems. In the case of a dynamical system such as an urban system we might for example pose questions about which types of behavior that are possible and how the system responds to changes in global parameters. Such models are not primarily created with the intention of predicting any urban system in particular but to explore basic system behavior such as catastrophes (in the mathematical sense), bifurcations, fractality, agglomerations, power laws etc. In addition, urban phenomena due to properties of historical dynamical processes such as lock-ins, industrial districts etc. have been explored. Examples of conceptual models in urban dynamics would include multiplicative growth models, models of correlated percolation, diffusion-limited aggregation, agent-based evolutionary models and neoclassical economic models e.g. (Alonso, 1964, 1972; Arthur, 1987; Axtell and Florida, 2001; Batty, 1991; Batty and Longley, 1994; Berry and W. L. Garrison, 1958; Dendrinos and Rosser, 1992; Fujita et al., 1999; Gabaix, 1999; Henderson, 1974; Lane, 2002; Makse et al., 1998; Manrubia et al., 1999; Marsili and Zhang, 1998; Reed, 2002; Rosser, 1998; Simon, 1955). As mentioned, a drawback of such models has been that the lessons that have been learned have not been easily carried over to applied models of the more topical type.

Dynamical models of urban evolution toward the topical end of the spectrum in urban growth modeling include primarily cellular automata, or at least, models that include a cellular automaton as an important sub model, and, agent-based models. Models of this type are typically designed for predicting the future of specific urban systems. Such models are often very complex, usually involving numerous sub-models in many layers and are subject to careful and systematic calibration of a large number of parameters. The use of cellular automata for urban growth modeling dates back over two decades to work by Tobler (1979) and was developed further subsequently by Couclelis, White, Clarke, Itami and others, e.g. (Andersson et al., 2002; Clarke and Gaydos, 1998; Clarke et al., 1997; Couclelis, 1985; Engelen et al., 1997; Itami, 1988; Torrens and O'Sullivan, 2001; White et al., 1997; Xie, 1996) and many recent applications. The cellular automata framework has been useful because of its flexibility: it is easy to incorporate any number of states and transition rules and cellular models can easily be layered hierarchically with a mix of micro and macro models. The drawback from a theoretical point of view is that the mapping between urban systems and cellular automata does not in itself carry with it much in terms of pertaining theory: not much may be learned about urban systems from the fact that they may be viewed as cellular automata and the framework is in this respect too flexible. The agent-based approach, such as the SIMPOP model (Sanders et al., 1997), shares the implementation flexibility of cellular automata but also the theoretical shortcomings. In addition, other approaches, more related to time series prediction in general than to urban evolution processes, have been used. Examples of such methods include artificial neural networks and logistic regression e.g. (Pijanowski et al., 2002; Yeh and Li, 2002) and Markov models e.g. (López et al., 2001). A class of other models that have seen extensive topical use are spatial interaction models, these are however not generally aimed at reproducing urban evolution.

Under the distinction between conceptual and topical models used here, the model presented below is to be viewed as a conceptual model that is constructed with the aim of being open-ended for elaboration into topical models. Some steps toward making the model more topical to an urban growth context are taken: we embed the basic network model into a cellular space and we make a distinction between different types of nearest-neighbor neighborhood types. The method of model construction that we have used and that we argue can be applied further is to start from the conceptual end of the spectrum and then moving step by step towards the more topical by adding more and more mechanisms. For each new set of additions we validate the behavior of the model. Additions should not destroy the desirable behavior of the simpler parent models, while at the same time, additions should add new meaningful behavior in agreement with empirical observations. In summary: the intention is to formulate a model that is i) reasonably stringent, ii) formulated on the micro level, iii) geographic (incorporates spatial interaction), iv) captures growth dynamics, v) is reasonably open-ended for additions and vi) is consistent with large-scale statistical properties of urban systems.

IV. THE CORE MODEL OF A COMPLEX EVOLVING NETWORK

Over the last decade, an increasing amount of theory for analyzing complex network models has emerged. The evolution of networks is understood as a sequential process of nodes and connections being added to the network and deciding to link to a particular subset of existing nodes according to some algorithm. That the probability of growth events are determined by the present system state makes complex networks an attractive choice for constructing models of the evolution of spatial economic systems. In this context, nodes can represent transportation nodes (e.g., airports, Internet routers), whole cities, or, as we prefer, land lots in a cellular space.

The main difference between complex networks and the way in which networks have usually been considered in connection to geography lies in their size. Properties of complex networks are studied as the networks become very large, considerably larger than what can conceivably be interpreted by human perception. Because of this, the study of complex network structure employs mathematical and statistical tools. Computer simulation is also central to this work as it enables implementation and testing of propositions and their effect numerically.

Point of departure is the graph-theoretical model of Barabási and Albert (1999), which has become a baseline model to explain uneven distributions as an outcome of growth dynamics within a network space. Essentially, the Barabási-Albert model elaborates on the principle of preferential attachment of the stochastic growth model by Simon (1955) in the context of networks. A network is a collection of nodes and connections between nodes and the term "complex networks" is commonly used for networks whose structure evolve over time due to the application of some mechanism by which new nodes are created and existing nodes are selected as end-points for new connections. Depending on the nature of this update mechanism and properties of the network (e.g. if it is directed or undirected) such networks come to have various properties as they grow. As will be outlined, one such property is the stationary power law distribution of node degrees found in the Barabási-Albert model. This model assumes that an increase in the connectivity (or degree) of a node in a network is a function of its current connectivity. More precisely, the probability $\Pi_i(t)$ that a new node connects to an existing node i, is linearly proportional to the degree $x_i(t)$ of node i at iteration t, i.e.:

$$\Pi_i(t) \sim x_i(t). \tag{1}$$

Several variations and extension of the initial model have been proposed as reviewed by Barabási et al. (2002); Dorogovtsev and Mendes (2002); Newman (2003). A simple, but fairly general version can be defined formally as follows: Consider a growing, enumerated set of nodes, $\{1, 2, \dots, N\}$ connected by undirected edges. As the network develops, new nodes and edges are added according to a stochastic model. There are no restrictions on multiple edges between two nodes and a node is allowed to have any number of connections to itself. The degree (number of connections) of node i is denoted by x_i.

The network is initialized by connecting n_0 nodes and at each iteration t the network is updated by the addition of one edge between two nodes that are chosen independently according to the following rules:

1. With probability q_1 the node is chosen uniformly between existing nodes. The probability of a node i to be selected is

$$\Pi_i^u = \frac{1}{N}. \tag{2}$$

2. With probability q_2 the node is chosen preferentially, which corresponds to the uniform selection of an edge end-point in the system and the subsequent location of its node. The probability of a node i to be selected is

$$\Pi_i^p = \frac{x_i}{\sum_j x_j}. \tag{3}$$

   This means that the probability is linearly proportional to the node degree.

3. With probability q_3 a new node is added to the network. This node will get a degree of 1.

The parameters q_1, q_2 and q_3, fulfills $q_1 + q_2 + q_3 = 1$ and are assumed to be constant during the evolution of the network.

When the growth rules are formulated, a number of questions can be posed about the properties of the network after a large number of iterations. The node degree distribution is one property that can be analyzed rather easily. It is possible to find an exact expression, using a master equation approach(Dorogovtsev et al., 2000), but for our purposes an approximate solution is sufficient. Such a solution can be obtained, by using a continuum formulation of the model. For node i, the time evolution of expected node degree follows

$$x_i(t+1) = x_i(t) + 2q_1 \frac{1}{N(t)} + 2q_2 \frac{x_i(t)}{\sum_j x_j(t)}, \tag{4}$$

where $N(t) = 2q_3 t$ is the expected number of nodes developed after t iterations. By using the continuous-time method introduced by Albert et al. (1999), we get that after sufficiently long time, the degree distribution approaches the form

$$P[x_i = x] \sim (x + B)^{-\gamma}, \tag{5}$$

with

$$B = \frac{q_1}{q_2 q_3} \tag{6}$$

and

$$\gamma = 1 + \frac{1}{q_2}. \tag{7}$$

The reason for including uniform growth is to ascribe for pure (additive) random growth that is not correlated with the present amount of activity in the particular land lot where growth occurs. In the context of urban growth, the argument for multiplicative growth (also referred to as Gibrat's Law), can be formulated in a number of ways with perhaps the most robust being based on a lack of information. It is fair to assume that growth often is generated as a direct result of existing activities. Now, if we do not know anything about the activity in two cities of sizes X and 2X then we must assume that the next growth event will occur in the first city with probability 1/3 and the second with probability 2/3. This argument, which has been used for cities most notably in models following Simon, should be applicable also to other areas inside which activity is located. Maybe the most natural relaxation of this growth rate assumption is to assume the mean and variance of the growth rate to be constant rather than the rate itself. This has been done by Gabaix to the end of achieving fast convergence to time-independent scale-free city sizes and is an important future extension also for the present model(Gabaix, 1999). In network terms, which holds greater generality than in this application, the model can be described as a linear combination between a random network (Erdös-Rényi network) and a scale free network (Barabási-Albert network). In a random network, the node selection mechanism is uniform per growing unit (land lot in this case) while in the scale-free network the selection mechanism is uniform per unit size of the growing units (per unit activity in the land lot here). The latter corresponds to the above argument for Gibrat's law and appears to be a realistic base model in many real-world situations including the previously cites systems that have been analyzed as scale-free networks.

According to Eq. (5) , this simple stochastic model can reproduce the power law distribution of degrees of nodes observed in many networks. Importantly, as shown by Barabási et al., both the assumption of growth and the assumption of preferential attachment are needed to reproduce the stationary power laws observed in empirical data on networks. If one assumes only uniform, and no preferential ($q_2 = 0$) attachment, the scale-free distribution is not reproduced. And, assuming a given non-growing ($q_3 = 0$) set of nodes, preferential attachment first produces a power-law, yet ultimately, the network evolves into a state in which all nodes are connected and the degree distribution tends to a Gaussian. It should be noted, though, that there are other network models that generate power law degree distributions without any of these mechanisms, see Newman (2003) and references therein.

The Barabási-Albert model thus, in a simple and reasonably robust manner, reproduces the scale-free degree distribution observed empirically in many complex networks. In the context of economic geography, the next step is to argue for the mapping between model and reality and to introduce additional "forces" involved in urban dynamics.

V. A COMPLEX NETWORKS MODEL OF URBAN EVOLUTION

Noting the fundamental network structure of the economy and the intimate relationship between this network structure and geographic distribution there are many ways of creating network models of economic geographic systems. Here we will present one such model where the aim has been to model general properties of geographic land value evolution. To obtain a model that is simple and that can be analyzed systematically it is always necessary to make simplifying assumptions. Rather than relying on assumptions of strong rationality and equilibrium analysis as in neoclassical economic models (Krugman 1991; Brakman et al. 2001), our assumptions are mainly statistical simplifications such as taking objects and interactions to represent large averages. This mode of simplification has been extensively used for similar purposes in other models including sociodynamics models due to Haken (1977), Weidlich (2000) as well as in some of the the agent based models and cellular automata models. In the following sections we will specify the model and briefly argue for the choices that were made.

A. Definition of objects and interactions

From assumptions of statistical averages over behaviors and activity types we construct a model that closely fits statistical empirical data of geographical land value distribution concurrently on several levels of abstraction: land value per unit area, land value per city and the relation between cluster area and perimeters. The parameters are approximated from empirical data and the model reacts smoothly to changes in parameter values. The model explains urban power laws as a consequence of multiplicative growth being sufficiently dominant to other types of growth and of a propensity for new land development to occur along the urban perimeter or roads. It also isolates the geographically heterogenous growth biases that give rise to cluster dynamics as a combination of transportation diseconomies and availability of infrastructure. These biases, which will be defined later in the text, are not strong enough to cause appreciable deviations from the power law properties of the Barabási-Albert network but sufficiently strong to cause geographic clustering.

Objects in a network model are represented as nodes and interactions as connections or edges between nodes. Agents are only implicitly modeled and their behavior is integrated in the selection of locations for growth. In the example used here we will use small fixed-size non-overlapping land lots as objects and trade streams between land lots as connections. Note that no explicit types of land uses are modeled and that this amounts to assuming that any number of land uses and goods/services exist. Given a connection endpoint of an unknown type all nodes are equally likely to have a demand or supply that makes it eligible for being the second endpoint. Concerning transportation, interaction probabilities or rates rather than explicit costs of transportation are used.

We note that profit (here considered before rent is paid) is generated as a consequence of trade between largely immobile specialized producers. Trade shall here be interpreted in a very broad sense, e.g. the interaction between a household and a workplace can be considered trade of labor for currency. This gives the basic network structure of the geographic economic system. Interactions, defined as trade streams, can simply be added and would constitute a valid definition also on the level of cities. On the level of lots, because lots here are per definition constant in area, the addition of a new connection does however have an additional important effect: it implies an increase in profit per unit area. If we assume that market pricing of land is a fast process compared to the process of urban growth, we can also invoke the left-over principle from urban economics to say that an increase of profit per unit area implies an increase in land rent. A new connection represents a new means of earning money in this location and because this behavior can be copied also by other agents, the land owner can generally charge rent for this. That is, if the present tenant does not want to pay more rent, then other potential tenants might. Land value, in turn, can be approximated by the present value of all future income streams stemming from rent. This allows us to compare empirical and simulated land values by taking node degrees as being proportional to market land value. It does, obviously, not allow us to reconstruct an explicit network.

B. Model formulation

When we introduce explicit spatiality in the model, the choice of the nodes in a pair can no longer be independent. This is handled by selecting one of the nodes before the other one. We call the first node the place of primary growth and the other one the place of secondary growth. The selection of these geographic locations are done either multiplicatively (proportional to current node degree) or additively (independent of current node degree).

Additive growth then becomes growth processes that are not directly correlated with present activity. For example, a road connecting two urban centers will cross sparsely developed areas and these areas will be subject to growth because of processes that have nothing to do with activity present along the road. The development (urbanization) of a previously undeveloped site must surely happen by an additive process, because there is nothing there to multiply. In the non-spatial model we treated this case separately, but now it turns out that we can handle node creation as additive growth on previously undeveloped sites. Note that node creation can also be termed "node activation" in this case since all nodes exists from the beginning – they are land lots that become "activated" when they are developed. See also Fig (1).

In this model we do not specify the type of activity that takes place at a site, only the activity level. Thus, activities are considered to be average activities and thus they all have the same probabilities for establishing interactions with each other. If we assume constant fractions q_1 and q_2 for additive and multiplicative growth respectively, with $q_1 + q_2 = 1$, the probability of selecting a node i multiplicatively in primary growth is

$$\Pi_i^{1,mul} = q_2 \frac{x_i}{\sum_j x_j}, \tag{8}$$

which is similar to the non-spatial case.

The probability of selecting a node i additively as a primary effect must be dependent on the availability of basic infrastructure. To keep it simple, we divide the sites into three categories - developed sites, perimeter sites and external sites. If we ascribe a weight a_i to each site (dependent on to which category it belongs) and pose the condition that the sum of all probabilities for additive primary growth should be q_1, we get

$$\Pi_i^{1,add} = q_1 \frac{a_i}{\sum_j a_j}. \tag{9}$$

The properties a_i of nodes are determined by a simple local infrastructure availability model that can be viewed as a cellular incarnation of cluster growth and birth mechanisms in cluster-based multiplicative models. Rather than as in cluster-based models only differentiating between cluster growth and cluster birth we here also want to capture the spatial patterns of clusters and spatial interaction between cells. The developed sites are defined to be the base case, and $a_i = 1$ for these. Note that taking a_i into account also for multiplicative growth would simply amount to a constant factor $1/\sum_j a_j$ since multiplicative growth only takes place on cells belonging to one infrastructure availability class (internal). Perimeter sites are all undeveloped sites adjacent to a developed site, and all these get

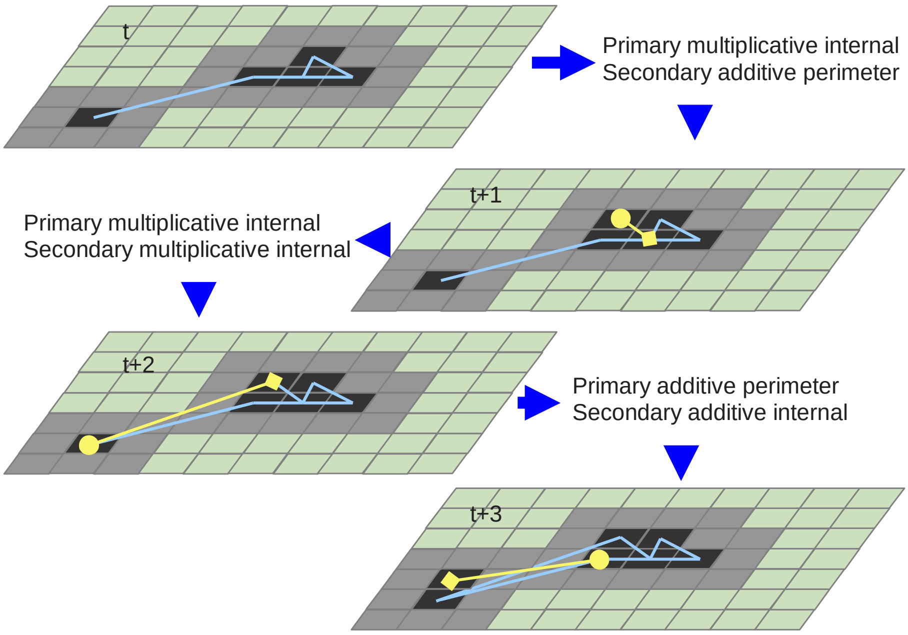

FIG. 1 A sample spatial network configuration with external, perimeter and internal nodes is updated stochastically by the addition of new connections. It is shown how nodes (cells) are selected at random according to the specified types of activity increases. For example, in the first update the first endpoint (diamond) is selected according to the multiplicative primary effect scheme and the second endpoint (circle) is selection with the secondary additive scheme.

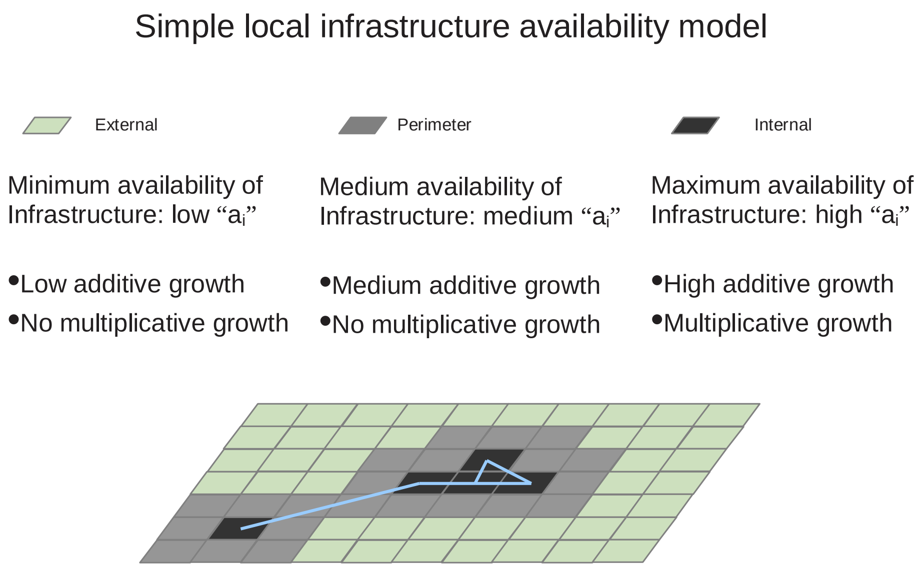

FIG. 2 A sample spatial network configuration with external, perimeter and internal nodes as well as connections between nodes is shown.

$a_i = b$, where b is a parameter. In most realistic cases $b < 1$, because there is, compared to a developed site, a less probability for an average perimeter site to have adequate infrastructure. Government growth control will also decrease b. See also Fig. (2).

External nodes are all nodes that are neither developed nor on the urban perimeter. In this case, usually only sites with direct access to roads can be considered for development. To avoid the (for the purposes of this model unnecessary) details in specifying a complicated model for the development of a road network, we assume that all external sites, with some probability have access to relevant roads and other infrastructure (thus effectively making it a perimeter node). This probability must surely grow with the system as more and more roads cross the hinterland, and we represent this by taking it proportional to the ratio between the number of perimeter nodes, $n_t^{(P)}$, and the number of external nodes, $n_t^{(E)}$ (The t-indices denote that these number change during the evolution of the system). We then get, for external sites, $a_i = b\epsilon \frac{n_t^{(P)}}{n_t^{(E)}}$, where ϵ is a constant parameter describing the relative density of infrastructure. (For this to be reasonable, we must assume that the lattice is not too crowded, i.e. $\epsilon n_t^{(P)} < n_t^{(E)}$.)

Now, when all three categories of sites have been considered, Eq. (9) can be written

$$\Pi_i^{1,add} = q_1 \frac{\delta_i^{(D)} + b\delta_i^{(P)} + b\epsilon \frac{n_t^{(P)}}{n_t^{(E)}} \delta_i^{(E)}}{\sum_j \left(\delta_j^{(D)} + b\delta_j^{(P)} + b\epsilon \frac{n_t^{(P)}}{n_t^{(E)}} \delta_j^{(E)} \right)} = q_1 \frac{\delta_i^{(D)} + b\delta_i^{(P)} + b\epsilon \frac{n_t^{(P)}}{n_t^{(E)}} \delta_i^{(E)}}{n_t^{(D)} + b(1+\epsilon) n_t^{(P)}}, \tag{10}$$

where $n_t^{(D)}$ is the number of developed nodes and $\delta_j^{(D)} = 1$ if node j is developed and $\delta_j^{(D)} = 0$ otherwise. The meanings of $\delta_j^{(P)}$ and $\delta_j^{(E)}$ are analogous to $\delta_j^{(D)}$, with P referring to perimeter sites and E referring to external sites. A site must belong to one and only one of these categories at the time, which means that for each j one and only one of $\delta_j^{(D)}$, $\delta_j^{(P)}$ and $\delta^{(E)}$ is equal to 1 while the two other equals 0.

Now, after the selection of the primary site, using one of the two mechanisms described above, the secondary site should be selected with a probability decreasing with distance from the primary one. To accomplish this, we define D_{ij} to be the spatial interaction strength between sites i and j. We have the restrictions $D_{ij} \leq 1$ and $D_{ij} = D_{ji}$.

The probability of secondary preferential growth at site i as a consequence of primary growth at site j is

$$\Pi_{ij}^{2,mul} = q_2 \frac{D_{ij} x_i}{\sum_k D_{kj} x_k}, \tag{11}$$

and analogical for secondary uniform growth, it is

$$\Pi_{ij}^{2,add} = q_1 \frac{D_{ij} a_i}{\sum_k D_{kj} a_k}, \tag{12}$$

and with the same site categories as for primary growth, this probability can be written

$$\Pi_{ij}^{2,add} = q_1 \frac{D_{ij} \left(\delta_i^{(D)} + b\delta_i^{(P)} + b\epsilon \frac{n_t^{(P)}}{n_t^{(E)}} \delta_i^{(E)} \right)}{\sum_k D_{kj} \left(\delta_k^{(D)} + b\delta_k^{(P)} + b\epsilon \frac{n_t^{(P)}}{n_t^{(E)}} \delta_k^{(E)} \right)}. \tag{13}$$

Several choices for D_{ij} are possible, but it is clear that the interaction strength should decay with increasing transportation costs. We have for the most part used

$$D_{ij} = (1 + cd(i,j))^{-\alpha}, \tag{14}$$

where $d(i,j)$ is the Euclidean distance between sites i and j. The non-negative parameters c and α controls the impact of spatiality.

1. Model summary

The model of trade relation end-point formation is implemented, as specified earlier, as a combination of additive and multiplicative growth, see Equations (8,10,11,13). Because of this, the model is a modification of the Barabási-Albert scale free network model, and theory for the analysis of such networks can therefore be applied. Costs of transportation and other distance-related diseconomies between two nodes i and j are captured by a bias, D_{ij} that makes long connections form at a slower rate than short connections. This distance bias modifies secondary additive and multiplicative growth rates, see Equations (11,13).

With this formulation we can keep the useful definition of an edge representing unit profit and node degree as land rent. This can primarily be viewed in two ways. Firstly, once a connection is formed it is always of unit benefit to its end-points but the further apart two lots are located the lower will the rate of connections forming between them be. This is because fewer and fewer potential trades will remain profitable when transportation costs are deducted. This is simply an alternative way of using an iceberg transportation cost logic: instead of connections becoming weaker over distance, their rate of formation tapers off. Secondly, larger than unit profit trade streams will have to be represented

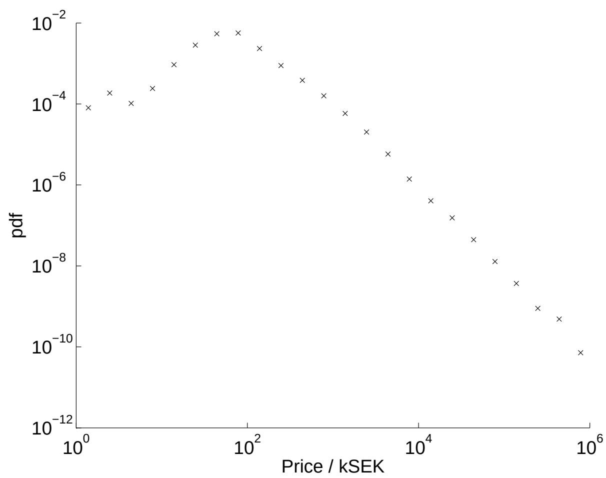

FIG. 3 A double-logarithmic histogram with exponentially binned Swedish land values. The prices are per Ha, and are based on 2.9 million taxing units, which give information on land values for a total 1.18 million Ha. A rough estimate for the power law exponent γ of the right part of the distribution is 2.1. We have chosen the price at the transition from the left to the right part of the distribution ($\sim$ 75kSEK/Ha) as the threshold for what is to be considered developed urban land.

as bundles of unit profit connections. Stretched out over a longer distance such that connection bundles will grow thinner as profits are reduced by transportation costs.

The model for availability of infrastructure is also simple and in addition local. We simply distinguish between three cases: internal, perimeter and external. Internal availability to infrastructure is maximal and used as unit. Perimeter availability applies to undeveloped lots adjacent to developed lots. External lots are lots that are undeveloped and whose geographic neighbor lots are also without development. External infrastructure availability are determined by the parameter ϵ and reflects the existence of an ambient infrastructure network serving long-range interactions. Such infrastructure spans undeveloped areas sparsely and may be "tapped into" thus making what will appear as new clusters of developed sites possible, see Equations (10,13).

C. Results

Agreement between model output and empirical data is important because failure to agree would allow us to disqualify the model or point to necessary modifications. However, empirical agreement does not validate the model in any stronger sense, because there generally exist many possible models that are capable of producing some sought-for behavior. Thus, the validation must be complemented with an argument for the mechanisms' mapping to real-world mechanisms. The predictions made from the model, a comparison between these predictions and measurements on empirical data and an argument for the choices and simplifications made in the model will be discussed in the following sections.

1. Analytical results

To simplify the analysis of the model it is useful to assume that development of new land (addition of an active node) takes place at a constant rate q_A (compared to other types of growth, not as a function of physical time). To motivate the assumption, let us consider the growth of developed clusters and their perimeters. It is true in simulations of the model, and it can be empirically verified (see Figures 6 and 8), that the cluster area distribution is close to a simple power-law with density function $f(A) \sim A^{-\beta}$, and that the relation between cluster perimeter size P and cluster area A has the form $P \sim A^{\lambda}$, with $\lambda < 1$. From this we observe that for the entire system of clusters we have

$$\frac{n_t^{(P)}}{n_t^{(D)}} \sim \frac{\int_1^\infty A^{-\beta} A^{\lambda} dA}{\int_1^\infty A^{-\beta} A dA} = \frac{\beta - 2}{\beta - \lambda - 1}, \tag{15}$$

assuming that $\beta > 2$ and $\lambda < 1$. This means that it can be expected that the ratio between the total numbers of perimeter nodes and developed nodes is reasonably constant. We now define q_1' as the fraction of primary activity

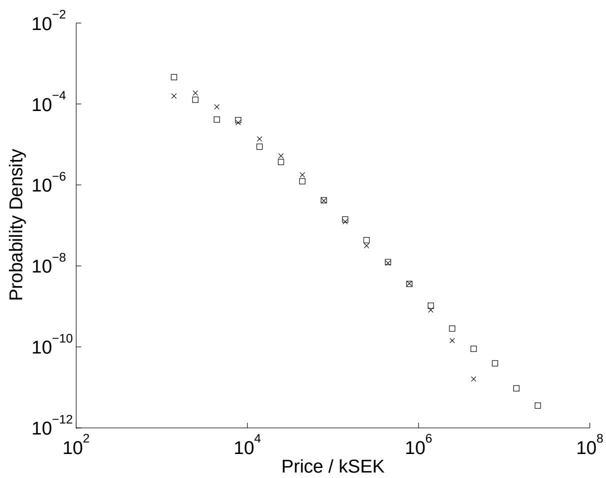

FIG. 4 A double-logarithmic histogram with exponentially binned empirical (×) and simulated (□) land values. The empirical land values are aggregated into 400m×400m sized cells.

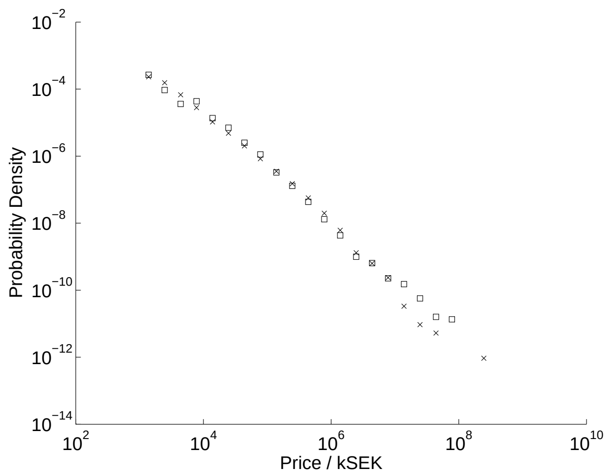

FIG. 5 A double-logarithmic histogram with exponentially binned empirical (×) and simulated (□) aggregated cluster land values. Empirical land values were aggregated to 400m×400m cells, and a threshold of 1425 kSEK/cell was applied. All contiguous (8-cell neighborhood) areas above this threshold were identified as clusters. These clusters are the same as for the results shown in Figures (6) to (9).

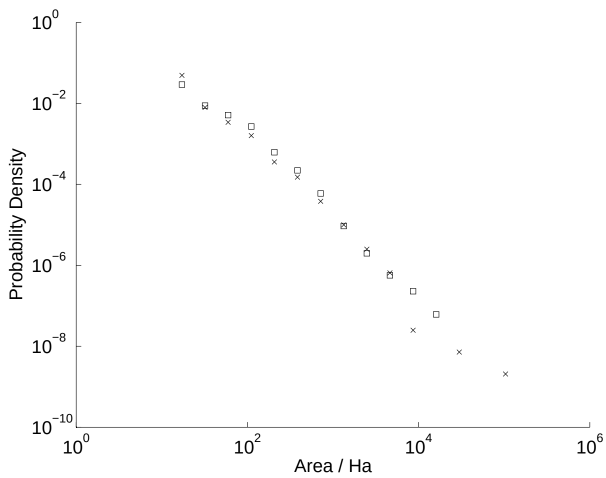

FIG. 6 A double-logarithmic histogram with exponentially binned empirical (×) and simulated (□) aggregated cluster areas. The clusters are the same as for as for the results shown in the other figures.

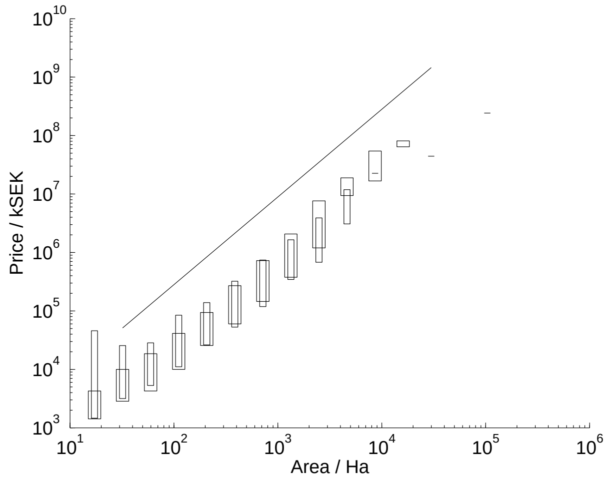

FIG. 7 This figure shows cluster area plotted against exponentially binned cluster prices for empirical (thin boxes) and simulated (broad boxes) results. The vertical interval of the boxes contains 90% of the prices in the bin. The reference line has a slope of 0.7. The clusters are the same as for the results shown in the other figures.

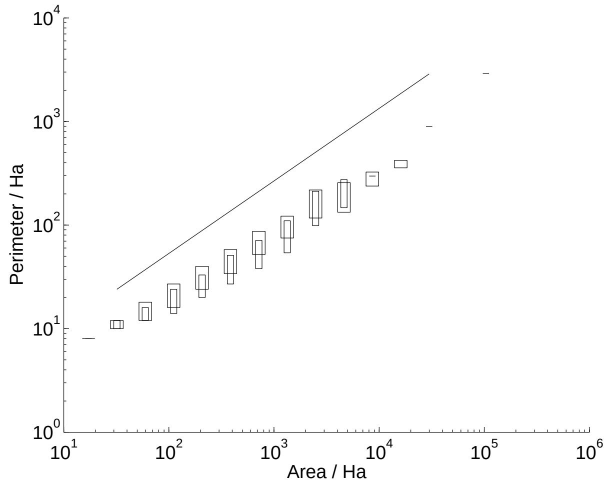

FIG. 8 This figure shows cluster area plotted against exponentially binned cluster perimeters for empirical (thin boxes) and simulated (broad boxes) results. The vertical interval of the boxes contains 90% of the perimeters in the bin. The reference line has a slope of 0.7. The clusters are the same as for the results shown in the other figures.

increments that occur on developed nodes under additive growth, and Eq. (10) gives

$$q_1' = \sum_i \delta_i^{(D)} \Pi_i^{1,uni} = q_1 \left(1 + b(1+\epsilon) \frac{n_t^{(P)}}{n_t^{(D)}} \right)^{-1}, \tag{16}$$

which, because of Eq. (15), is approximately constant. This means that the rate of primary node activation, $q_A = q_1 - q_1'$, also can be considered constant.

It can be argued that expected secondary growth behaves very similar to primary growth, even when the impact of spatiality is strong (Andersson et al., 2003). This means that the time evolution of expected activity on a developed site i can be approximated by

$$x_i(t+1) = x_i(t) + 2q_1' \frac{1}{n_t^{(D)}} + 2q_2 \frac{x_i(t)}{\sum_j x_j(t)}, \tag{17}$$

which is similar to the non-spatial model described earlier. Thus, after long time the degree distribution can be

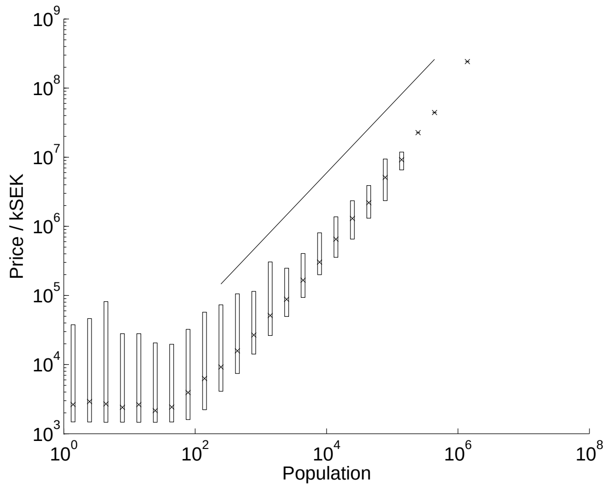

FIG. 9 This figure shows empirical cluster population plotted against exponentially binned empirical aggregated cluster prices. The vertical interval of the boxes contains 90% of the perimeters in the bin, and the crosses indicate the medians of the prices in the bins. The reference line has a slope of 1.0, which indicates that there is a near linear relationship between cluster price and population, for clusters with a population larger than 100. The comparison was carried out by first identifying geographical clusters of high land values (above the threshold 1425 kSEK/cell) and then comparing their accumulated population content with their accumulated value. The clusters are the same as for the results shown in the other figures.

expected to approach a generalized power law, $P[x_i = x] \sim (x+B)^{-\gamma}$, with

$$B = \frac{q_1'}{q_2 q_A} = \frac{q_1'}{q_2(1-q_2-q_1')} \tag{18}$$

and

$$\gamma = 1 + \frac{1}{q_2}. \tag{19}$$

2. Endogenous node fitness

The basic growth mechanism in this model is stochastic multiplicative growth which makes it closely related to models such as Simon's, Gabaix's and Gibrat's models. This means that there is no need to invoke agglomeration economies, external economies or transportation costs for explaining the presence of urban hierarchies: the urban hierarchy can essentially be viewed as a consequence of stochastic multiplicative growth (land value per unit area) and city edge growth (land value per city). Agglomeration economies, local increasing returns to scale and transportation costs are important because we know that they exist in the real system and a model must be able to include such forces and still be valid. In other words, although the mentioned forces may not be responsible for observed hierarchies or agglomerations, they are important for explaining other urban phenomena and their presence must be consistent with the existence of hierarchies. Hence, the question may not be not whether transportation diseconomies and agglomeration economies cause skewed distributions but rather if the statistics of the simple model "survives" the introduction of such forces. It will be discussed later that local variation in growth rates may be crucial in a secondary way because it enables fast convergence from initial configurations that are not power law.

Knowing the criteria for scale free node distributions in growing networks of the proposed kind to be that growth is asymptotically multiplicative we want to be able to analyze the complete node growth rates to investigate whether it still fulfills these criteria when some additional forces have been included. To facilitate this sort of analysis it is useful to observe what we here call "endogenous node fitness". Fitness, for short, specifies the full growth rate of the nodes and allows us to observe in what way it deviates from a pure multiplicative process.

We can calculate the expected value of the growth Δx_i of any site by summing over the probabilities of all primary and secondary growth events at i,

$$E[\Delta x_i] = \Pi_i^{1,uni} + \Pi_i^{1,pref} + \sum_j \left(\Pi_j^{1,uni} + \Pi_j^{1,pref}\right)\left(\Pi_{ij}^{2,uni} + \Pi_{ij}^{2,pref}\right). \tag{20}$$

By separating the terms containing a multiplication with x_i from the ones which do not, the expression can be written as a combination of multiplicative (preferential) and additive (uniform) growth,

$$E[\Delta x_i] = \frac{\eta_i x_i}{\sum_k \eta_k x_k} + \zeta_i \tag{21}$$

with

$$\frac{\eta_i x_i}{\sum_k \eta_k x_k} = \Pi_i^{1,pref} + \sum_j \left(\Pi_j^{1,uni} + \Pi_j^{1,pref}\right) \Pi_{ij}^{2,pref} \tag{22}$$

and

$$\zeta_i = \Pi_i^{1,uni} + \sum_j \left(\Pi_j^{1,uni} + \Pi_j^{1,pref}\right) \Pi_{ij}^{2,uni}. \tag{23}$$

For large enough x_i, additive growth will always be negligible compared to multiplicative growth. This means that we have asymptotic preferential growth, which has shown to be a sufficient criterion for obtaining a scale-free network (Krapivsky et al., 2000). Thus, the factor with most potential to notably affect the type of degree distribution is the site- and time-dependent η_i. The structure of Eq. (21) shows that there is a strong resemblance between η_i and the concept of node fitness(Bianconi and Barabási, 2001; Ergün and Rodgers, 2002). This means variation in η_i might cause the degree distribution to be a sum of power laws with different exponents. But in all simulations of the model, η_i turns out to fall in an interval narrow enough for multiscaling to be negligible.

In the appendix it is shown that the multiplicative fitness can be written as:

$$\eta_i = q_2 \left(1 + \sum_j \left(\sum_k x_k \Pi_j^{1,uni} + q_2 x_j\right) \frac{D_{ij}}{\sum_k D_{jk} x_k}\right). \tag{24}$$

What is important to note, is that it is not particularly dependent on the value of x_i.

From the above equations and figures we can learn a number of things about the model. First of all, since we can say that a model where growth is asymptotically multiplicative will yield scale free statistics we also know the types of functional forms that we can allow η_i and ζ_i to have if we want to retain power law statistics. Any content that is added, meaning forces that influence location decisions, will show up in these functions. We may add any content that we wish and still maintain scale free behavior as long as node growth remains essentially multiplicative. Now, if we assume that real-life urban systems are in fact growing networks we may also analyze known deviances from scale-free behavior using the fitness concept. One known deviation is that of a capital outlier: that all or at least the majority of a county's cities conform to the rank-size rule but that the largest city is "too large". In network terms what has taken place is that the nodes of the largest city have grown faster than in proportion to their size. This can translate to an η_i that is strongly dependent on x_i or to strong positive growth correlation between nearby activities of certain types. The reasons for such growth rate deviations can be sought among the commonly proposed types of agglomeration and scale economies which are not represented in this model. Another known type of deviation is found in economies that are or recently were planned such as Russia where instead the large cities are "too few". In such cases, a large amount of growth might not have been multiplicative in the first place.

3. Model validation

If we want the model to reproduce some important statistical properties of a certain urban system, then most of the parameters in the model can be roughly estimated from empirical data. For instance, because ϵ controls the ratio of external development to perimeter development, and external growth for the most part gives a remaining new cluster, we have the approximate relationship

$$\frac{\epsilon}{1+\epsilon} \simeq n_t^{(C)}/n_t^{(D)}, \tag{25}$$

where $n_t^{(C)}$ is the number of clusters. The exponent γ in the distribution of land prices gives information about the size of q_2 (and q_1) via Eq. (19). The perimeter parameter b controls the ratio of perimeter development and internal additive growth, and it can then be determined by

$$q_A \simeq \frac{n_t^{(D)}}{\sum_i x_i(t)} \tag{26}$$

and with the use of Eq. (16) we get

$$b \simeq q_A \left((q_1 - q_A)(1+\epsilon)\frac{n_t^{(P)}}{n_t^{(D)}} \right)^{-1}. \tag{27}$$

The fact that the current state of the system constrains the value of these parameters does not mean that they are purely empirical in nature. They are aggregated measures of fundamental properties of the current economic system (and region) of interest, and in the general case they may change with time. If the model should be used for prediction, instead of explanation as is the case here, then more elaborate ways of estimating current (and future) parameters are needed. The small number of parameters that are used in the present model appear to be sufficient for reproducing some large-scale regularities of the system but quite naturally not for prediction on any finer resolution.

The spatial parameters c and α in Eq. (14) reflect the statistics of transport characteristics of the economic configuration. Their effect on the statistical properties of the system are somewhat subtle and thus they are not as easily estimated from data as the parameters mentioned above. Anyhow, results are robust to their exact values and the functional form of D_{ij}. A variety of parameter values as well as linear, exponential and constant functional forms have been used and alterations appear to impact primarily the internal structure of the network but not to a large degree the statistical properties investigated here. This is also indicated by the properties of Eq. (24).

The data used for all empirical results is based on a database delivered by Sweden Statistics that covers estimations of the market value of all land in Sweden (2.9 million data points). The database is constructed from street addresses of taxing units, mapped to coordinates. The estimated land values of all units with coordinates within a 100m×100m cell are summed to become the reported land value of this lot. In Figure (4), the distribution of these land prices (1.18 million lots) are shown. It is evident that a large part of Swedish land values are power law distributed, an observation also reported for Japan (Kaizoji, 2003). To connect the empirical land price data to our model, we need to choose a threshold for what we consider a developed site. We choose this threshold to be 75 kSEK/Ha (10 SEK ≈ 1 USD), which is about at the transition between the two regions in the empirical histogram. Some taxing units are larger than one Ha and roads and water separate developed sites from each other, which are effects that do not appear in the model. To get around this, and to make the data set somewhat more manageable in size, we aggregate the land values to 400m×400m cells. A new developed threshold is chosen to be 1425 kSEK/(16Ha) to get the same ratio between developed and total land area as in the 100m×100m case (2.0%).

Cluster measurements was obtained by identifying clusters using this threshold for developed sites. To identify clusters in simulated and empirical data we use a computer program that masks away all data points below a threshold value and then treats all contiguous (8-cell neighborhood) areas as clusters. With this procedure, 7747 empirical clusters were identified.

Simulation of network evolution produces a network configuration in which the nodes are characterized by their location on the lattice and the connections that they have to other nodes. Node degrees (the number of connections to a node) in such configurations are taken to correspond to land value by the mapping that was previously explained in Sec. (V.A). From the mentioned properties, additional output is computed such as spatial clusters of high node degrees which corresponds to urban places. The theoretical properties of the model suggests that it can reproduce the observed power law distribution of urban land values to the extent that this mapping is correct. To further validate the model, some statistical measures that faithfully capture the spatial configuration of the node degrees are needed. Probability distributions of cluster observables, such as areas, perimeters and aggregated prices are such measures. Using the model presented in this paper we obtain agreement between simulated and empirical statistics for a range of these higher-order structures in successive orders of upward causation (cells to clusters), see Figures (4) to (8). Figures (4) to (8) reflect data from the same simulation run of the model. To reproduce the power law exponent in the empirical distribution of land values, $q_1 = 0.2$ was used. The parameters $b = 0.34$ and $\epsilon = 0.18$ were then estimated from empirical cluster data as described above. The spatial parameters were $c = 0.1$ and $\alpha = 2$. Investigation of sensitivity shows that exponents and proportions change slowly and smoothly with all parameters. This was investigated by carrying out simulations for a wide range of parameter values. Furthermore, the values of the measured quantities, being stationary properties of the network under growth, re-appear robustly regardless of random seed used. As will be discussed more in-depth later in the text, the present model with constant multiplicative rates of node growth converge at a power law node degree distribution only when grown from a small initial configuration, or, obviously, from a configuration that already is power law distributed. For convergence to take place from a wider range of seed configurations, variations in local growth rates need to be introduced in a fashion analogous to Gabaix model(Gabaix, 1999). A square grid of 1600×1600 cells (each cell representing a 16 Ha square) was used to match the land area of Sweden (41,093,400 Ha). The number of iterations was $267,956$, to give the final simulated configuration the same total price as the aggregated empirical price of developed sites (763,675,011 kSEK). In comparisons with empirical land values, the model land values were taken as $1425x_i$ kSEK (degree 1 then corresponds to the threshold land value for developed sites).

The results are not trivial consequences of the node degree (land value) distribution; it is perfectly possible to arrange the developed cells into any system of clusters. The same is true for the relation between cluster area and perimeter. It can also be noted that since the growth model is based on random multiplicative growth the model does not explain the presence of a stationary hierarchical organization as being caused by increasing returns or agglomeration economies. Such effects can be incorporated and that they may enable to model to capture additional properties of the system. However, in agreement with other multiplicative growth models, the results indicate that these concepts are not central to explaining skewed distributions in urban systems.

In the tails of the distributions, there are some deviations between empirical and model data. In Figure (4) there are more very high priced land in the model than in the empirical data, and in Figures (5) to (8) it is evident that the largest empirical clusters have larger areas than in the model. None of these results are very surprising. Regarding the high land values there are two reasons - firstly, there is a great uncertainty in the estimations and reporting of market value for the most expensive central urban land, and secondly there might be congestion factors, not present in the model that prevent a too high degree of activity concentration, which should show up as a cutoff in the empirical land price distribution. For the deviations of large cluster areas, there are also several reasons. One is that the largest cluster (Stockholm) is somewhat of an outlier in the distribution, with an aggregated price 5.5 times, and an area 3.1 times that of the second largest cluster. Another reason is that coastal areas with high land value around the large cities extend their areas in a way not captured by the present model. Also, the positive feedback from the availability of transportation networks around large cities are not fully present in the model, because D_{ij} is taken as constant, which of course is a simplification. Such forces count amount the previously discussed deviations from pure multiplicative growth and because such forces are known to exist we must also expect that real-life distributions deviate from simple power laws to some extent.

In Figure (9) we demonstrate that for Sweden, aggregated cluster population and aggregated cluster land value are linearly related. This could be interpreted as an indication of that population growth might be understood as a response to the growth of the economic network.

The simulations we carry out are extrapolations from a near-empty initial configuration and thus we do not aim to reproduce the configuration of economic activities in Sweden but instead a system that is statistically similar to said configuration in some important observables. We are at this point only interested in large-scale regularities such as power law exponents, and, needless to say, it can hardly be expected that a model with only a handful of parameters can make any detailed predictions about a real system. Predicting a specific configuration (such as extrapolating Sweden from the year 2000 to 2005) brings historically path dependent phenomena into focus and would demand a considerably more detailed model incorporating coast lines, roads, additional forces and so on and such a model is beyond the scope of this paper. According to the logic previously argued for, reconstructing a system that is consistent with large-scale statistical regularities such as those here studied logically precedes the implementation of a detailed model for scenario prediction. Details of the mentioned kind would only serve to confuse the model at this stage of analysis.

In its present incarnation the model converges very slowly to a power law distribution unless the initial configuration is very small. However, if local variations in growth rate is introduced in a fashion similar to what is done in Gabaix model, then convergence to a power law, at least for the largest clusters, takes place rather quickly. As previously mentioned, Gabaix relaxed the assumption of constant rates of growth to rates of growth with constant mean and variance. Growth rate variation, referred to as “shocks” by Gabaix, are in Gabaix model introduced at the city level which is natural because this is the lowest level in that model. In the network model, however, growth rate variation can be introduced both at the level of network nodes and at the emergent scale of clusters. That is, the cluster membership of nodes can be tracked during network growth and biases of growth rates can be applied to nodes both based on their node- and cluster identity. Preliminary results indicate that introduction of variation on any of these levels has the sought-for effect. Additionally, a network model would also make it possible to investigate the effect of more realistic shock dynamics such as propagation of shocks through the trade network. Endogenous variation in growth rates exist in the present model but they are generally very small, see the discussion of endogenous node fitness in Sec. (V.C.2). In fact it might turn out that cited forces that affect location choice indeed have an important role to play in the generation of hierarchies although in a perhaps somewhat surprising and indirect way – they can conceivably be the “impurities” to simple multiplicative growth that is necessary for rapid convergence. For variation of the required type to be endogenous to the model, the model needs to be extended in ways that remain to be determined. We leave a thorough investigation of this important aspect to subsequent papers.

Another possible source of error when comparing to an empirical data set relates to the fact that the real-world system is in fact only a part of a larger system. It might for example be such that Sweden lend itself fairly well to such a comparison because the largest region (Stockholm) and the second largest region (Gothenburg) are located in a fairly central position in the data set. Furthermore, the land borders of Sweden, which are mainly towards Norway, runs through areas of comparably low development. If, on the other hand, some continental European country would have been analyzed, the fact that its borders would likely run through more densely developed areas might have caused problems. Still, it is clear that also a large number of the connections in the real-world trade network of Sweden does have its end-points outside of Sweden. Furthermore, it must be suspected that such connections are not evenly distributed over the activities in the system, relating also to the problem of averaging over area: large cities with ports and airports may have a larger proportion international trade than smaller cities.

D. Discussion

In formulating the model we have sought for some fundamental structure and logic of the geographic economic system. The basic logic behind multiplicative growth models – that growth occurs at a rate that is proportional to the present amount of development – seemed to be a sound starting point: if we do not know anything about the activities in the system, a city with population X must have half the probability of a city with population 2X to be the location of the next growth event. Additional knowledge about the forces acting in the urban system (if available) then perturbs this multiplicative base model of location choice. This logic, to the extent that it can be applied to cities, must also be applicable also to other contiguous geographical areas. So, it should be possible to apply multiplicative growth also to a cellular model where the cells are fixed-size non-overlapping areas. As the next step to arrive at a geographical formulation we concluded that the most important force that shapes the system geographically is the cost of transportation (in a wide sense). Related to the cost of transportation is also the availability of infrastructure – transportation costs in areas not connected with transportation networks are of course extremely high. That is, there is a spatial correlation between units of activity: trading activities will tend to be in closer proximity than activities that do not trade and new development will tend to take place in locations that are adjacent to infrastructure (on the perimeter of cities and along roads). This imposes a constraint on multiplicative growth and therefore threatens the power law distribution of node sizes. On the other hand, such forces are likely crucial for higher-order dynamics such as the arrangement of development into clusters with certain properties rather than some other spatial distribution of power law land values. Another important aspect of trade, and one that we also concluded is fundamental to the dynamics of the system, is that activities appear not in isolation but in trading constellations. That is, if activity increases in one location there will be a corresponding increase in activity elsewhere representing the balance between demand and supply. For simplicity, we presently choose to consider strictly only pairs of activities. That is, when growth occurs it occurs in pairs, and, because of costs of transportation, the trading pair will more often be near each other than far apart and the trading end-points will locally be adjacent to infrastructure. Our candidate for a fundamental model of the geography of economic activity can therefore be summarized as follows: the location of new activity is for the most part multiplicative, trade involves strictly more than one activity, transportation costs causes a bias toward short distances between trading activities and a propensity for new development to take place in connection with infrastructure. At this point, the connection to the Barabási-Albert model of growing scale-free networks was obvious since it simulates the evolution of a system by multiplicative addition of connections between nodes. To apply it to the problem at hand, it was however necessary to extend it in a number of ways.

The model however still needs ad hoc additions to be able to generate realistic output. A model with only multiplicative growth has some obvious and fatal flaws with maybe the most obvious being that undeveloped cells can never be developed. Development of new nodes is of course crucial for simulating urban growth because without it the urban perimeter would never grow and no new cities would appear. To this end additive growth is introduced which was explained earlier in the article as various effects that disturb pure multiplicative growth. For example, roads will bestow high market potential to areas that are without nearby development. Also, if some opportunity for growth appears in an area there might be a considerable freedom of choosing the exact location within a larger area and an activity might then choose to settle in a nearby unoccupied lot rather than outbidding a present urban activity.

The present Euclidean model of distances is, needless to say, another simplification that potentially could introduce systematic errors in the statistical regularities here studied. Basically, the present transportation model amounts to Euclidean distance with modifiers for transportation availability in external, internal and perimeter areas. There are two main reasons for why we have selected a very simple characterization of transportation. Firstly, the statistical behavior of the system, in the properties we have observed, is not very sensitive to the distance decay of interaction rate. This relates of course to the fact that transportation costs are not viewed as causing the observed regularities – to the contrary the focus is on showing that it does not destroy them. In the light of this, stronger historical transportation constraints may have meant that earlier urban systems conformed less to the rank-size rule than presently. Also, it would mean that recent and future improvements of transportation technology will make urban systems more and more power law distributed: lower transportation costs translates to smaller disturbances to pure multiplicative growth. This is however highly speculative since there are also many other important factors that affect local growth rates. Secondly, a more complex representation of transportation, while not buying much in terms of prediction, would mean that additional and potentially risky assumptions must be made. By the principle of parsimony, we choose the simplest possible model that captures the phenomena we wish to address at this stage. A better transportation model could however shed more light on for example the internal structure of clusters, so this is definitely still an interesting way in which to elaborate the model in the future. See Figure (10) for a qualitative illustration of the impact of using a simplistic transportation network model. It can be noted in this figure that the most prominent visual difference between simulation and reality is that real urban clusters are largely arranged in streaks following the extent of major roads.

The micro dynamics of the present model is formulated such that interactions on all length scales follows the same rules. In reality there might be systematic deviances from such behavior both on the short and the long scales. That is, the present set of rules might best capture the behavior of trade on some medium scale. Short-scale interactions, which would shape the intra-urban structure, might behave in some other way and very long-range interaction, responsible for inter-urban dynamics, in yet another way. Such effects are not captured by the present model. This relates to a comment on the present model by Pumain (2004) where she points out that there exists important differences between inter-urban and intra-urban dynamics.

Finally, it must be noted that complex network provide a solution to the problem of delineation in urban studies. In models in general an important aspect of selecting objects resides in their properties as "individuals". This subject has been discussed for example in biology in relation to models of selection based on objects on a higher and lower level than organisms(Ghiselin, 1974; Gould, 2002). To be a proper individual an object should have a definable temporal beginning and end and in between it should remain identifiable and integral. That is, it should not fuse with other objects or split up other than in very predictable ways (such as a biological cell dividing) or dissolve so that its identity becomes hard to define, neither should it pass in and out of existence. The reason for this is that we must be able to define mechanisms that govern the dynamics of the system and we must be able to maintain properly defined measures. This is a problem that exists for multiplicative growth models formulated on the level of cities but that is circumvented by the present definition of nodes and fixed-size land lots. Cells, or nodes, remain exactly the same throughout the entire simulation.

The cellular formulation also allows us to efficiently tackle the problem of defining and measuring the quantities under study. Because cells stay the same throughout simulation time we may also in a strict way study their properties: number of connections per unit area, number of cells per cluster and so on. Furthermore, since the data set with which we compare the model's output is also in a cellular format this allows us to use exactly the same analysis algorithms on both sets. In other cases when model output and real world data is compared the simulation output is delimited in one way and the empirical data set may have been delimited in a number of different and potentially even unknown ways. The problem of delimitation in studies of urban growth is discussed by for example Pumain (2004).

VI. COMPLEX NETWORKS IN ECONOMIC GEOGRAPHY IN A WIDER PERSPECTIVE

Models of urban growth can loosely be grouped into economic and non-economic models. These two tracks of development represent widely different perspectives on the urban system and they have progressed more or less in isolation from each other. Economics models are generally hard to apply and often lack realism because they are constructed with mathematical tractability as their primary design parameter. Non-economic models are generally constructed with realism in the foreground but this comes to the cost of lacking a common analytic framework. While economic models rely on strong rationality assumptions and equilibrium analysis to achieve mathematical tractability, non-economic models generally rely on less restrictive assumptions to achieve the same thing. To understand the strengths and weaknesses of complex networks models in this spectrum, we discuss a number of economic and non-economic models.

Urban economics stems from Ricardian rent theory via the monocentric city model of von Thünen to its later extension for urban land uses by Alonso as well as influences from other schools of thought such as central-place theory. As mentioned, economic models are based on rationality as a behavioral assumption. This and other mentioned assumption simplifies the models considerably and forms the basis for the mathematical machinery behind neoclassical economics.

Systems of cities models following Henderson (1974) models city sizes as being affected by centrifugal and centripetal forces. Factors that act to create agglomeration, centripetal forces, are referred to as economies of agglomeration and urbanization and include economies of scale, thick labor markets, information spill-overs, sharing of transportation infrastructure and so on. Factors that act against agglomeration, centrifugal forces, are for example congestion and immobile inputs. Furthermore it is argued that external economies are often sector specific so there is a tendency of production specialization on the city scale. The ultimate size of a city represents and equilibrium between the centripetal and the centrifugal forces and the point at which this equilibrium occurs is determined by the type of production that the city specializes in. The Henderson model, like many other urban economic models, identifies the forces that are at play but because system states are viewed as points of equilibrium it does not say much about growth dynamics. Furthermore, the Henderson model is not geographic: although there is cross-city migration there

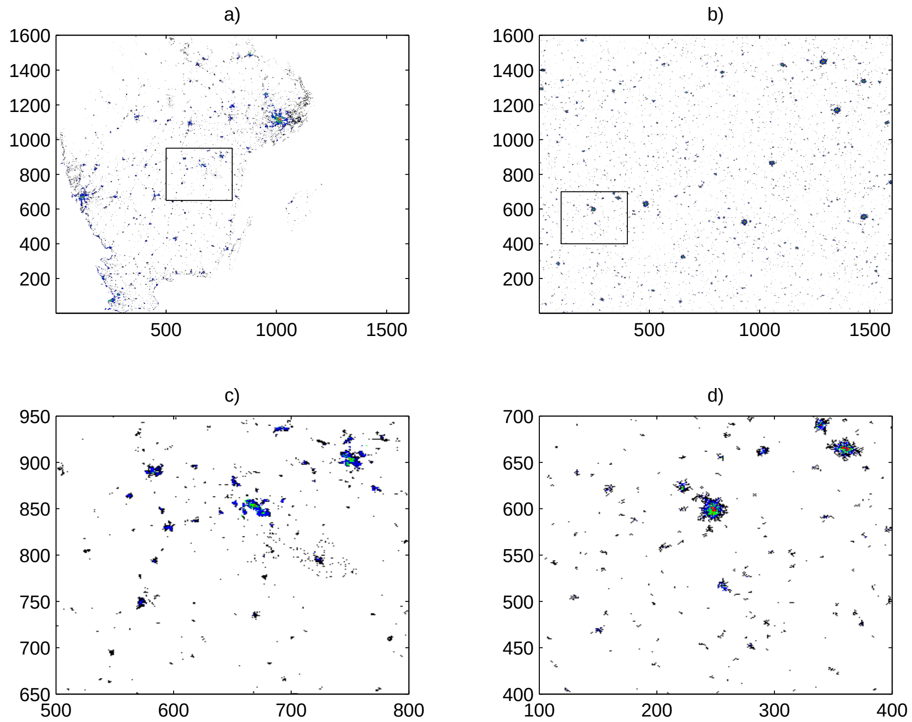

FIG. 10 Above, empirical (a and c) and simulated (b and d) geographical configurations of land values are shown in two levels of zoom. The colors represent value per 400m ×400m cell as follows; white: $0 - 1424$ kSEK, black: $1425 - 4999$ kSEK, blue: $5000 - 29999$ kSEK, green: $30000 - 99999$ kSEK, red: ≥ 100000 kSEK. It can be clearly seen that the empirical configuration is arranged around natural and anthropogenic structures such as roads, rivers and coastlines. These being absent in the simulation, the clusters become more idealized, but still exhibit a similarity in both intra and inter-urban shape and composition. As is verified statistically earlier in the paper, the statistical properties of the simulated and the real system under the observables that we have used are highly similar. Also, as pointed out in the text, no difference is made between intra-urban and inter-urban dynamics in the model.

is no trade between cities and the geographical locations of cities therefore becomes meaningless. Also, there are some problems in bringing the system to equilibrium from various starting points because the model can not explain how cities grow organically from a small size. Here the model relies on the existence of large agents more or less capable of founding entire cities at once and the realism of this as a universal process can be questioned.

The new economic geography models due to Krugman and others is a successful attempt to extend the neoclassical framework to a geographical setting. This is done with the expressed intent to provide a stronger theoretical basis to regional science. The core model of the new economic geography (Krugman, 1991) combines mobile firms and workers in a model with transportation costs and increasing returns to scale within firms. From this, it can be derived that, with transportation costs sufficiently low, firms and workers will cluster in cities to minimize costs of production and transportation. The model has shown to be sufficiently rich as to allow for others extensions(Brakman et al., 2001; Fujita et al., 1999). The new economic geography models achieve analytic tractability but this does come to a rather severe cost of realism. Because of this, these models have not exclusively been greeted warmly by geographers and others with an interest in urban evolution. What is more, the new economic geography models do not address urban growth, but are better understood as an evolutionary game in which economic actors (firms and consumers) decide rationally on a location by taking into account the expected decisions of all other agents. Following this reasoning, the resulting equilibrium is technically an evolutionary stable Nash equilibrium (Brakman and Garretsen, 2003). Thus, although the new economic geography is stringent, formulated on a micro level, geographic and extendable in many directions, it does not include growth dynamics as such.

The geographical pattern of land use changes only slowly and as a consequence of this, the state in which it may be found can not be understood as an equilibrium. Systems whose states represent equilibria are typically characterized by the concurrent action of fast dynamics and slow dynamics or exogenous parameters: the slow dynamics or static exogenous parameters determine the location and types of equilibria and the fast dynamics causes the system to converge on an equilibrium state. In urban dynamics however, the forces that affect the evolution of the system typically chance faster and not slower than the state of the system. Thus, equilibrium analysis appears to be a poor choice for explaining geographical distributions of land use.

The complex network approach is in some respects closer to the older base-multiplier concept: we can select an area and measure how many connections that have both end-points inside this area and how many that only have one end-point there. We can also compute the expected rate of growth and the expected rate of formation of connections of the previously mentioned type. However, because we do not differentiate between seller and buyer we can for example not in the present formulation tell what is export from what is import. To make a network model more unified with base-multiplier analysis, a directed network could be used. Furthermore, it is also possible to introduce cascading effects similar to those used in base-multiplier analysis: that growth events and removals are propagated through the connections in the network.

Complex network models, in the present application or any other application to economic or social geographic systems simulates the formation of interactions in a specific or an abstract sense. Because of this, there exists a connection between complex network models and spatial interaction modeling. For example, it can be noted that the distance bias function that is used (Eq. 14) bears strong resemblance to a gravity function with the exponent as a parameter as introduced by Huff (1963). The probabilities of persons from a region i shopping some another region j in Huff's retail model is also almost identical to the calculation of multiplicative secondary effects in the present model, see Eq. (11). Spatial interaction models in dynamical incarnations are primarily equilibrium models and thus not aimed at explaining questions about the evolution of urban systems but the connection is nevertheless highly important for a number of reasons. First of all, according to Gibrat's Law or any related assumption of multiplicative growth, there is a direct relation between measures of activity and interaction and measures of the rate at which activity and interaction grows. Secondly, because complex network models simulate the formation of connections it is also possible to observe the quantities typically used in spatial interaction models such as connection strength or number of trips between regions and amount of activity in regions. Furthermore, a wide range of probabilities and relative densities can be observed in relation to cells and regions simulated with a complex network and we are often interested in the relation between macrostates and microstates. This means that entropy measures that are commonly used in spatial interaction modeling could find their use also in relation with complex networks(Wilson, 1970, 1967). This need not only be the case for urban systems but also in a cross-scientific perspective for other systems that are analyzed using complex networks. Entropy has also been used by Curry (1964) to explain the rank size rule, although he viewed the urban hierarchy as an equilibrium state of maximized entropy.

Another set of models to which the complex network approach is related is a family of 'complexity' approaches that explain urban growth and urban patterns using conceptual models such as diffusion-limited aggregation (DLA), correlated percolation, reaction-diffusion models and multiplicative cluster growth models. As discussed earlier, the elementary model by Simon is truly foundational in this respect, and reproduces the rank-size rule for cities, yet is not geographic formulated on a very aggregated level. Moroever, such models can not address morphological aspects of urban systems. DLA models due to Batty address the fractal properties of urban patterns but not other statistical properties of the urban system such as the rank-size rule(Batty and Longley, 1994). Also, the micro formulation based on random walkers appear not to be very realistic and the site of the urban core must be specified at the beginning of the simulation. The correlated percolation model of Makse and Batty et al reproduces a number of power laws and appears more realistic in its micro formulation(Makse et al., 1998). Also here the simulation starts from a configuration with an already existing gradient centered on the cite of the central business district. Manrubia and Zanette introduced a reaction-diffusion model of population density evolution(Zanette and Manrubia, 1998). This model combines predictions of characteristics of clusters and geographical patterns at the same time. The model captures some fundamental properties of urban growth such as population migration and multiplicative growth and has some statistical similarities with the present model. However, no secondary growth effects are used (growth events do not occur in spatially correlated pairs), no long-range interactions take place and the micro mechanisms are very abstract.

The important strength of the present complex network model is that it is easier to see how it can be extended into more topical incarnations than many of the conceptual models discussed in the previous paragraphs. Also, the network model reproduces a wide range of statistical properties that earlier conceptual models have reproduced only in isolation. For example, it reproduces the area-perimeter relation, the land value per unit area and the cluster area distributions at the same time. This means that it is consistent with both Simon-type models of cluster growth and with pattern formation models such as correlated percolation at the same time. Also, although the micro dynamics are abstract they are intended to capture a fundamental network characteristics of the system. This is in contrast with for example the reaction-diffusion model of Manrubia and colleagues (Manrubia et al. 1999; Zanette and Manrubia 2000), where micro mechanisms do not clearly map to corresponding real-life mechanisms.

A. Future extensions

Linking microscopic mechanisms to macroscopic structure is a defining theme in modern complexity-related research. For social and economic systems, the importance of this task is self-evident. Policies must be directed at the small changes that incrementally form the evolutionary economic system over time since changing the macrostructure would be exceedingly expensive and uncertain. At the same time, it is the macroscopic states of the system, e.g. geographical distributions of various measures, that most greatly affect the agents' welfare and motivate policy programmes. Thus, exploring how agent actions, and policies directed at regulating them, map to macroscopic states is central to the task of planning and policymaking.

The crucial asset of complex networks as a basic model for evolution in economic geography is that it can be applied over a wide spectrum of uses: from abstract models whose primary objective is to pave way for theoretical models to arbitrarily complex models for scenario prediction and exploration. It is not hard to see how multiple commodities and services can be used and how correlations between such types of commodities and services may be introduced by specifying types of activities that take specified input and produce specified output. Also, many of the global properties of complex networks such as those measured in this paper and related papers are the result of basic properties of the evolution mechanisms and there is no reason to believe that these would not carry over well to more models with more complexity in these mechanisms. For example, the requirement on a Barabási-Albert scale free network is that growth is asymptotically multiplicative. This is not a very constraining requirement and it leaves much room for elaboration on the specifics of the growth events. This includes incorporation of other important forces in urban dynamics such as increasing returns. This means that models can be constructed in a direction from the general to the special without the special case losing the desired properties arising in the general model.

With respect to application to land use patterns, the complex network approach is compatible with cellular automata models of urban development, which has been discussed earlier in this text. It must be noted that a spatial network model is in effect a network model embedded in a cellular space. Thus, the appeals of cellular automata in geographic modeling – combining form and function in a common framework – must also be considered to apply in this case. In addition, the network model adds to that list also the crucial aspect of transportation and allows for considerably more rigorous and open ended analytical and statistical analysis. The objects in the model can find additional uses in more complex setups. Nodes can be assigned properties belonging to land areas such as slope, terrain, land improvements and their age, value, which activities they may accommodate and so on. More elaborated transportation models may also be used, maybe most importantly to have a more realistic estimation of distance than the Euclidean distance used presently. The growth of such transportation systems may also be added as a sub model. In doing so, interactions may be differentiated technologically, based on the spatial scale over which they occur.

VII. APPENDIX

Because of the normalization in Eq. (21), the exact expression for η_i can be chosen in different ways (i.e it can be varied by any multiplicative constant). To make individual η_i independent of the size of the system, we choose $\sum_i \eta_i x_i \sim \sum_i x_i$:

$$\eta_i = q_2 + \sum_j \left(\sum_k x_k \Pi_j^{1,uni} + q_2 x_j \right) q_2 \frac{D_{ij}}{\sum_k D_{jk} x_k} = q_2(1 + A_i + B_i), \tag{28}$$

with

$$A_i = \sum_k x_k \sum_j \frac{D_{ij} \Pi_j^{1,uni}}{\sum_k D_{jk} x_k} \tag{29}$$

and

$$B_i = q_2 \sum_j \frac{D_{ij} x_j}{\sum_k D_{jk} x_k}. \tag{30}$$

By using the symmetry $D_{ij} = D_{ji}$ and observing

$$\sum_i A_i x_i = \sum_i x_i \sum_k x_k \sum_j \frac{D_{ij} \Pi_j^{1,uni}}{\sum_k D_{jk} x_k} = \sum_k x_k \sum_j \frac{\Pi_j^{1,uni}}{\sum_k D_{jk} x_k} \sum_i D_{ij} x_i = \sum_k x_k \sum_j \Pi_j^{1,uni} = q_1 \sum_k x_k \tag{31}$$

and

$$\sum_i B_i x_i = q_2 \sum_i x_i \sum_j \frac{D_{ij} x_j}{\sum_k D_{jk} x_k} = q_2 \sum_j x_j \frac{\sum_i D_{ij} x_i}{\sum_k D_{jk} x_k} = q_2 \sum_j x_j \tag{32}$$

it can be verified that

$$\sum_i \eta_i x_i = q_2(1 + q_1 + q_2) \sum_i x_i. \tag{33}$$

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